Date: March 13, 2014
Teacher: Tuğba Özcan
Number of Students: 15
Grade Level: 9
Time Frame: 45 minutes

Trigonometric Ratios in Right Triangle with Unit Circle

1. Goal(s)

- Students will know the relation between unit circle and the basic trigonometric ratios defined by the angles of a right triangle.

2A. Specific Objectives (measurable)

- Students will be able to define sine, cosine and tangent ratios for an acute angle.
- Students will be able to solve right triangle problems by correct selection and use of sine, cosine and tangent ratios.
- Students will be expected to use geometric concepts and properties with spatial reasoning to solve problems in fields such as engineering.
- Students will use trigonometric ratios to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

2B. Ministry of National Education (MoNE) Objectives

- Dik üçgende dar açıları trigonometrik oranlarını tanımlar ve uygulamalar yapar.
- Bir açının sinüs, kosinüs, tanjant ve kotanjantı dik üçgen üzerinde tanımlanır.
- Dik üçgende; 30°, 45° ve 60° nin trigonometrik oranları özel üçgenler yardımıyla hesaplanır.
- Birim çemberi tanımlar ve trigonometrik oranları birim çember üzerindeki noktanın koordinatlarıyla ilişkilendirir.
- Sadece 0° ile 180° arasındaki açıların trigonometrik oranları birim çember yardımıyla hesaplatılır.

2C. NCTM-CCSS-IB or IGCSE Standards:

- Students should use trigonometric relationships to determine lengths and angle measures (NCTM).
- Students should use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (NCTM).
- Students should use geometric models to gain insights into, and answer questions in, other areas of mathematics (NCTM)

- Students should explain and use the relationship between the sine and cosine of complementary angles (CCSS).

- Students should use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems (CCSS).

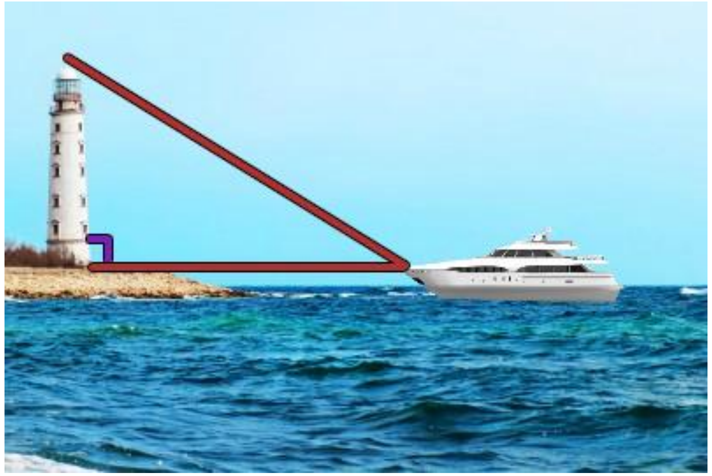
3. Rationale
   - Students will need to know the trigonometric ratios to solve problems in other mathematics topics
   - Students should use the relation between the trigonometric ratios and unit circle to solve problems in, and gain insights into, other disciplines such as physics and other geometry topics and other areas of interest such as engineering and architecture.
   - Students will need to know trigonometric ratios about how they are applicable to everyday life.
4. Materials
   - Board markers
   - Computer and Projector
   - Worksheets, exploration sheets (there are 15 students in class and copy each of them for 15 students)
5. Resources
   - TED 9th grade geometry book
   - McDougal Littell Geometry Book
   - PhSchool Teacher resources www.phschool.com/math
   - Hease and Herris Mathematics for the International Students SL and HL
   - Hease abd Herris Cambridge International Mathematis
   - http://www.eba.gov.tr/video/izle/389459072782c4df04544bc5c7dbab4c009792d09c001
6. Getting Ready for the Lesson (Preparation Information)
   - Copy activity sheets for each student. There are 15 students in the class.
   - Before writing, open the projector and show the activity sheet on the board explain students the instructions given in the activity sheet
   - Monitor students while they are working and help them if they need
7. Prior Background Knowledge (Prerequisite Skills)
   - Students will be expected to state and apply the Pythagorean Theorem.
   - Students will be expected to know the basic properties of a right triangle
   - Students will be expected to familiar with what the circle is and how it is drawn in coordinate system

Lesson Procedures

Transition: Good morning everyone. Today, we're going to study trigonometric ratios in right triangle.

8A. Engage (5 minutes)

- Show the pictures by using projector. The pictures are related to the daily life examples of trigonometric ratios in a right triangle.



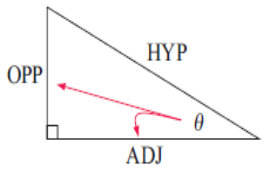
- Tell the story about the history of the lighthouse and ships.

"With the marine development, the lighthouse are becoming to guide the ships through the long vacations. There are different types of lighthouses with different functions. The captains do some calculations according to the lighthouse features. Sailors usually need to know the distance a ship from the land. But for this, they need to calculate the height of lighthouse and the angle between the ship and the line connecting the sea level. Hence, we obtain a right triangle whose vertexes are formed by the ship, lighthouse and land. So we can find the distance to the land of ship by the trigonometric ratios of a right triangle."

Transition: Let's discover how we can find this kind of distance by using the trigonometric ratios for the lighthouse to guide a ship

B. Explore (15 minutes)

- Remind the students trigonometric ratios of a right triangle.(You can use the information below)



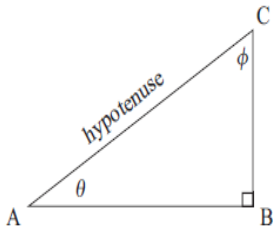
For the right angled triangle with angle θ:

- the **hypotenuse (HYP)** is the longest side
- the **opposite (OPP)** side is opposite θ
- the **adjacent (ADJ)** side is adjacent to θ.

Given a right angled triangle ABC with angles of θ and ϕ:

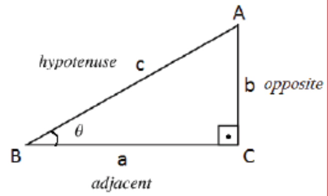
For angle θ, BC is the **opposite side**
AB is the **adjacent side.**

For angle ϕ, AB is the **opposite side**
BC is the **adjacent side.**



4.4.2. Trigonometric Ratios in Right Triangle

The trigonometric functions *sine, cosine, tangent* and *cotangent* can be defined according to the ratios of the sides of a right triangle.



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c}$$

$$\tan\theta = \frac{opposite}{adjacent} = \frac{b}{a}$$

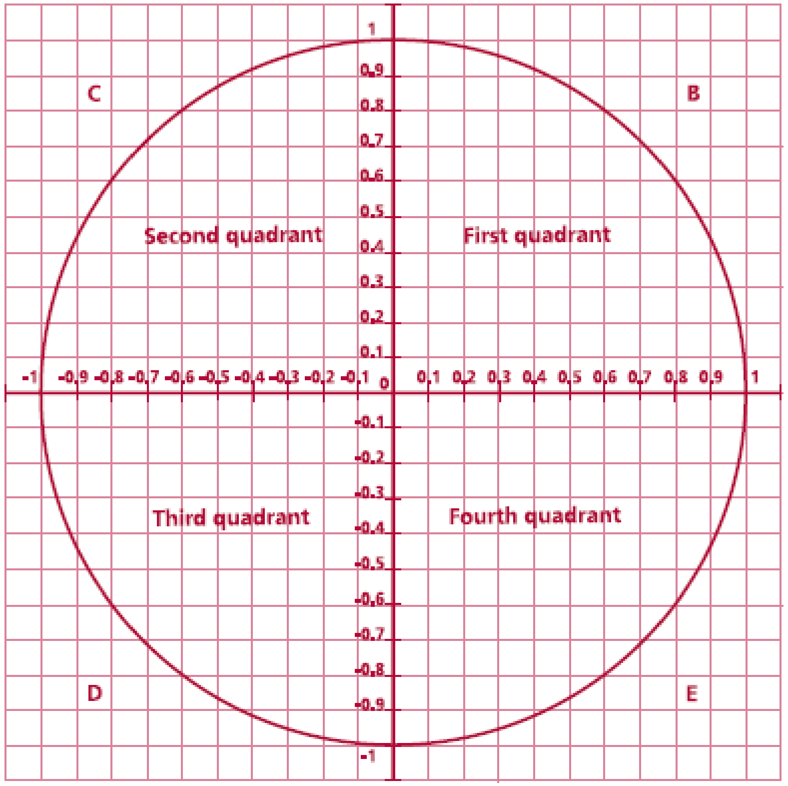
$$\cot\theta = \frac{adjacent}{opposite} = \frac{a}{b}$$

- Then distribute the exploration sheet and give a clear instruction of it
- Wait 10 minutes for the student to explore the idea of unit circle in the worksheet
- Next want the students to complete the table below

Exploration Sheet

TED ANKARA KOLEJİ

Exploration Sheet



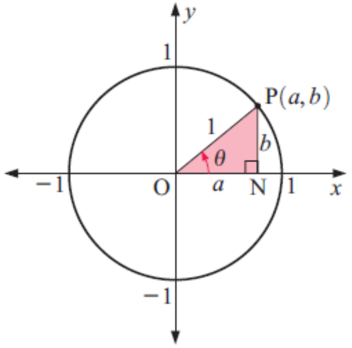
1. On the unit circle above mark an angle of 30° in counter clok wise. And label the point which intersects with the circle as P.
2. Drop perpendicular line from P to the x-axis to construct a right angle triangle, centered at (0,0).
2. What is the length of hypotenuse?
3. What is the length of opposite?

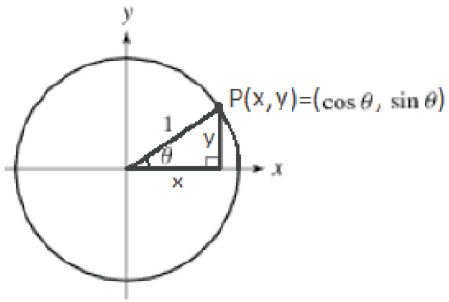
4. What is the length of the adjacent?
5. Using trigonometric ratios, not a calculator, calculate the sin30°, cos30° and tan30° cot30°.
sin 30°= cos 30°= tan 30°= cot30°=
6. Compare these with the values of the x and y coordinates of P. What do you notice about the x and y coordinates of P and the trigonometric functions sin 30°, cos 30° and tan 30°?
7. Check the answers using calculator.
sin 30°= cos 30°= tan 30°= cot30°=
8. Now, mark an angle of 60° in counter clock wise. And label the point which intersects with the circle as Q. Read the x and y coordinates of the point Q, where the terminal ray intersets the circumference.
9. Drop perpendicular line from Q to the x-axis to construct a right angle triangle, centered at (0,0).
10. What is the length of hypotenuse?
11. What is the length of opposite?
12. What is the length of the adjacent?
13. Using trigonometric ratios, not a calculator, calculate the sin60°, cos60° and tan60° cot60° .
sin 60°= cos 60°= tan 60°= cot60°=
14. Compare these with the values of the x and y coordinates of P. What do you notice about the x and y coordinates of Q and the trigonometric functions sin 30°, cos 30° and tan 30°?
15. Check the answers using calculator.
sin 60°= cos 60°= tan 60°= cot60°=
16.. Now, mark an angle of 90° in counter clock wise. And label the point which intersects with the circle as R. Read the x and y coordinates of the point R, where the terminal ray intersets the circumference.
17. Drop perpendicular line from R to the x-axis to construct a right angle triangle, centered at (0,0).
18. What is the length of hypotenuse?
19. What is the length of opposite?
20. What is the length of the adjacent?
21. Using trigonometric ratios, not a calculator, calculate the sin90°, cos90° and tan90° cot90° .
sin 90°= cos 90°= tan 90°= cot90°=
22. Compare these with the values of the x and y coordinates of P and Q. What do you notice about the x and y coordinates of R and the trigonometric functions sin 90°, cos 90° and tan90° cot90°?

Transition: let's talk about what you find

C. Explain (20 minutes)

- Then draw a circle on the board and say "it is a unit circle which is a circle with center O(0,0) and the radius is 1 unit."

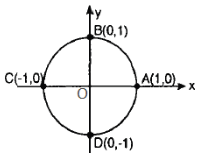




- Explain the unit circle property. You can use the definition below

4.4.3. Unit Circle

The circle with a radius of 1 unit, centered at the origin of a rectangular coordinate system is called the unit circle.



Let θ be the angle measured from the positive x-axis to the terminal side, then the point P on the unit circle where the terminal side intersects the unit circle is defined to be P(cos θ ,sin θ).

That is, the first coordinate (abscissa) of a point on the unit circle is cos θ and the second coordinate (ordinate) is sin θ .

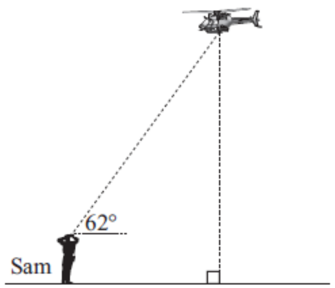
Transition: I am distributing the worksheet, make a group of three and solve these problems

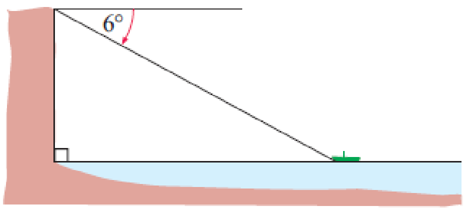
D. Extend (10 minutes)

- Distribute the worksheets and give clear instruction
- Explain the rubric of the worksheet and want the students to solve these problems according to the rubric
- Use the rubric to evaluate students work

WORKSHEET

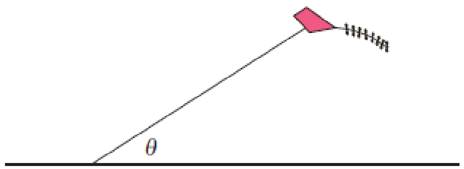
A model helicopter takes off from the horizontal ground with a constant vertical speed of 5 m/s. After 10 seconds the angle of elevation from Sam to the helicopter is 62^o. Sam is 1.8 m tall. How far is Sam's head from the helicopter at this time?

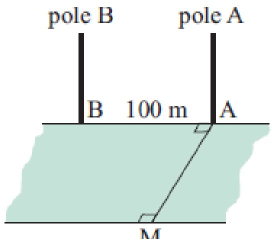




From a vertical cliff 80 m above sea level, a fishing boat is observed at an angle of depression of 6^o. How far out to sea is the boat?

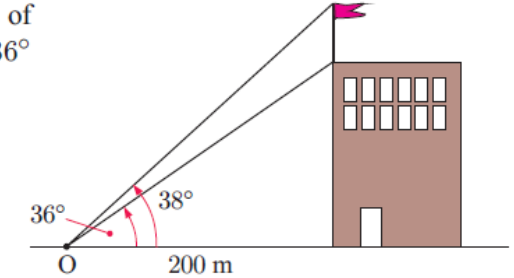
A kite is attached to a 50 m long string. The other end of the string is secured to the ground. If the kite is flying 35 m above ground level, find the angle θ that the string makes with the ground.





A man, M, positions himself on a river bank as in the diagram alongside, so he can observe two poles A and B of equal height on the opposite bank of the river.
He finds the angle of elevation to the top of pole A is 22^o, and the angle of elevation to the top of pole B is 19^o.
Show how he could use these facts to determine the width of the river, if he knows that A and B are 100 m apart

From an observer O, 200 m from a building, the angles of elevation to the bottom and the top of a flagpole are 36° and 38° respectively. Find the height of the flagpole.



- Encourage whole students in the lass to solve every problems on the worksheet
- By oral questioning get students to solve the problems
- Ask for justification and clarification from students
- Encourages students to explain concepts and definitions in their own words

Transition: I am evaluating your performance in group work according to the rubric

E. Evaluate (during the whole class)

- Assesses students' knowledge and skills through oral questions.
- Evaluate each students performance in group work by using the rubric while they are studying the worksheet
- Observe the students during the lesson and check each student's answer.
- Want the students to write a short paragraph about what they learn today and where they can use this information in their daily life

9. Closure & Relevance for Future Learning

- Ask students is there any points not understood.
- Then say them ”Ok, thank you so much for this enjoyable lesson”
- Give the students their homework
- State the next topic of the lesson

10. Specific Key Questions:

- How can you define sin of an angle in a right triangle?(knowledge)
- What did you notice in this problem?(analysis)
- How can we generalize the way that we observed?(synthesis)
- What if we chance the triangles then what would the ratios be the same?(application)
- What do you think about unit circle ?(comprehension)
- Why the such important trigonometric ratios in a triangle for us?(application)
- How is the trigonometric ratios in a triangle with unit circle?(synthesis)

11. Modifications

- If students cannot remember previous lesson, give them some clues.
- If students do not give answer to your questions, wait 20 seconds more.
- If the students cannot share any idea the teacher will wait twenty seconds more.
- If the students need more time while they solve the exercises the teacher will give one more minutes.
- If the students cannot solve the exercises the teacher will come back the definitions or previous examples and explain again
- If students confuse about the answer clarify it by another way
- If they cannot solve the problems or did not understand, want another student to explain their friends how they used which method