Date: May 8, 2014
Teacher: Tuğba Özcan
Number of Students: 18
Grade Level: 11 IB-SL
Time Frame: 45 minutes

Power of a point with respect to a circle

1. Goal(s)
   - Students will be able to understand the concept of power of a point with respect to a circle.

2A. Specific Objectives (measurable)
   - Students will be able to realize the different position of a point with the circle.
   - Students will be able to realize the meaning of power of a point.
   - Students will be able to demonstrate the general formulas of power of a point.
   - Students will be able to prove the power of a point for each position of the point.
   - Students will be able to realize power is always positive if the point is in the exterior region of the circle.
   - Students will be able to realize power is always negative if the point is in the interior region of the circle.
   - Students will be able to realize power is always zero if the point is in the circle.
   - Students will be able to apply their knowledge about the power of a point on the questions.

2B. Ministry of National Education (MoNE) Objectives
   - Çemberin bir noktasındaki teğeti ile ilgili teoremleri ispatlar ve uygulamalar yapar.
   - Bir çember ile bir doğrunun birbirlerine göre konumunu belirler ve uygulamalar yapar.

2C. NCTM-CCSS-IB or IGCSE Standards:
   - Students should use trigonometric relationships to determine lengths and angle measures (NCTM).
   - Students should use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (NCTM).

3. Rationale
   - Students should solve problems involving civil engineering, astronomy, and meteorology by the help of a knowledge of power of a point in a circle.
   - There are many examples on the books this topic was asked many times in the OSYM exams. The examples will be given to the students as homework.

4. Materials
   - Board markers
   - Computer and Projector
   - Worksheets, exploration sheets (there are 18 students in class and copy each of them for 18 students)

5. Resources

- Glencoe Geometry Book
- Euclidian Geometry Book Hease and Harris
- TED Power-point resource
- MEB 11 grade book, 2013
- Turkish Education Association Publications, Geometry 11.

6. Getting Ready for the Lesson (Preparation Information)

- Copy worksheets for each student. There are 18 students in the class.
- Before the lesson, make sure the projector and the power point are working or not
- Monitor students while they are working and help them if they need.

7. Prior Background Knowledge (Prerequisite Skills)

- Students will be expected to state and apply the basic properties of circles.
- Students will be expected to familiar with terms such as chords, secants and tangents of the circles.
- Students will be expected to know the equations of the circles.
- Students will be expected to know how to find the radius of circle by using its equation.

Lesson Procedures

Transition: Hello Guys! As you know this is the second time we do the lesson together. This time we will do geometry together. I learned nearly all of your names but maybe I can confuse so I want you to put your name cards on your desks again.

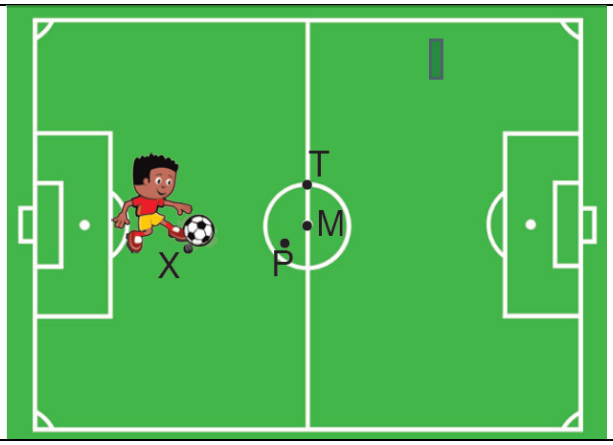
8A. Engage (1 minute)

- The teacher will mention about the power of a point topic does not include in IBDP-HL curricula, so they are not responsible the topic in IB examinations. But it includes in MEB curricula and there are many examples that they were asked in OSS and LYS examinations. For this reason they are responsible the topic for school examinations and university entrance exam.

Transition: So, today we will talk about the powers of a point. Now, I will distribute one worksheet to you. You will work with your peers and You have 5 minutes. Let me explain what you need to do for accomplishing this worksheet. There is a football field photo on the sheet and the story is written on the sheet. You need to read carefully. Do not forget to underline the important terms on it, it is useful for you. After that follow the other question on the worksheets and accomplish them.

B. Explore (10 minutes)

- The teacher will reflect the football ground photo.
- The teacher will distribute the worksheets and want students to do the sheet.



In the image of a football field, the midfield and the soccer are seen. The midfield is a circle which is centered at the point M with radius r. Let the difference between the square of the distance from the soccer to the center of the circle, which is M, and the squre of radius be K(X).

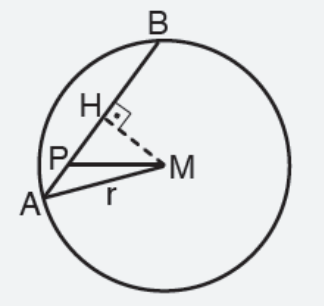
a) Write the K(x) function.

b) Examine the function of K(x), when the footballer is inside the midfield and outside the midfield

c) Examine the function of K(x), when the footballer is on the circle.

d) By starting the point X, the soccer comes to the point T on the circle with a linear motion. By considering this, make a connection between the K(x) at the point X and the square of t

e) When the footballer is in the midfield, on the point of K, write the K(x) function according to |AP| and |PB|.



f) Find the distance from the point X to the point T.

- The teacher will ask students what they realized. She asks:
  "What positions can be a point of X ? " (Analysis)
  "What happens to the P(x) function when the point of X is on the circle?" (Analysis)
  "Why you find 0 when the X is on the circle?" (Analysis)
  "Can it be true for all points on circle?" (Comprehension)

Transition: Now, let's understand the concept of the power of a point with respect to a circle.

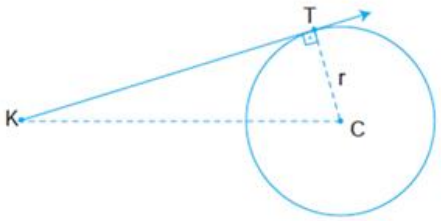
C. Explain (15 minutes)

- The teacher will open the TED power point presentation and explain the theorems and examples step by step.
- The teacher " You said that there are three different possibilities for the position of the point X. Let's label the point X with the K."
- The teacher will reflect the first situation on the board and prove all of them by asking the steps to the students. She also distributes all reflections as a sheet to the students and wants students to take note on it.
- "Let's write the power of the point of K for each of them"

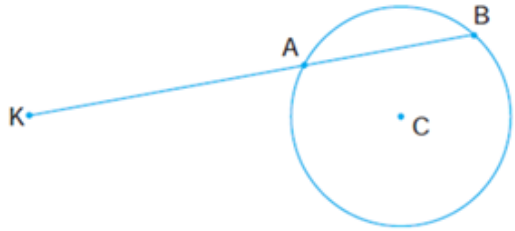
Power of a Point

There are three different possibilities for the position of the point K.

1) If $K(x_0,y_0)$ lies in the exterior region of the circle then

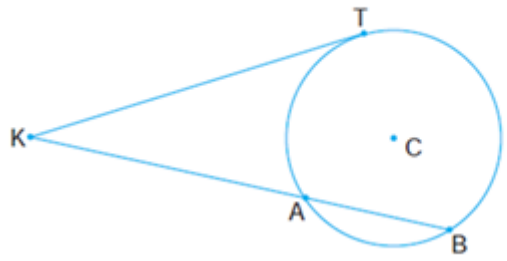


T is point of tangency.



A and B are points on the circle.

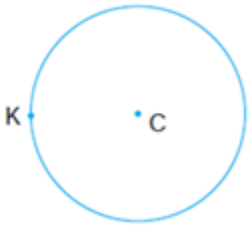
Notice that



$$\|\overrightarrow{KT}\|^2 = \|\overrightarrow{KA}\|.\|\overrightarrow{KB}\|$$

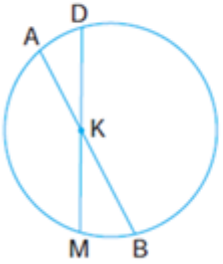
- The teacher wants students to realize power is always positive if the point is in the exterior region of the circle and so the teacher asks:
  " What did you realize about when the point is in the exterior region of the circle?" (Analysis)
- The teacher wants students to be written the P(x) function for the second one.

2) If $K(x_0,y_0)$ lies in the interior region of the circle then



- The teacher reflects the third one and wants students to share their idea about how they can prove the given.

3) If $K(x_0,y_0)$ lies in the interior region of the circle then



$$P(K) = \|\overrightarrow{KC}\|^2 - r^2 = -\|\overrightarrow{KA}\|.\|\overrightarrow{KB}\| = -\|\overrightarrow{KD}\|.\|\overrightarrow{KM}\|$$

- The teacher asks students:
  "Did you realize anything on these equations $P(K) = -|\overrightarrow{KA}|.|\overrightarrow{KB}| = -|\overrightarrow{KD}|.|\overrightarrow{KM}|$?"

- The teacher wants students to realize power is always negative if the point is in the interior region of the circle and so the teacher asks:
  " What did you realize about when the point is in the interior region of the circle?" (Analysis)
- The teacher reflects the conclusion part and wants students to look at the first one and explain what is says and why that can be true.

Conclusion

1) The power of point $K(x_0,y_0)$ with respect to the circle $x^2 + y^2 = r^2$ is

$$P = x_0^2 + y_0^2 - r^2$$

2) The power of point $K(x_0,y_0)$ with respect to the circle $(x-a)^2 + (y-b)^2 = r^2$ is

$$P = (x_0 - a)^2 + (y_0 - b)^2 - r^2$$

3) The power of point $K(x_0,y_0)$ with respect to the circle $x^2 + y^2 + Dx + Ey + F = 0$ is

$$P = x_0^2 + y_0^2 + Dx_0 + Ey_0 + F$$

Transition: Now, I shall distribute you worksheets and you will do it by individually. You have 10 minutes, I shall move around the classroom and if you want to ask anything you need to raise your hand. Okay!

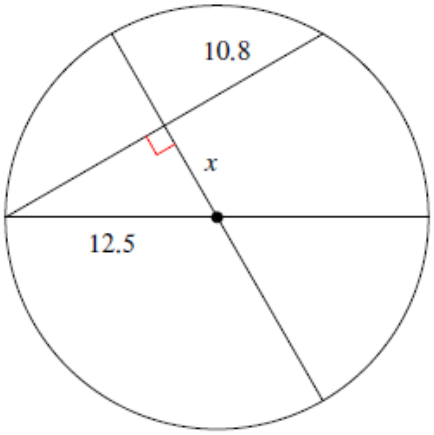
D. Extend (10 minutes)

- The teacher distributes the worksheets and observe students' works.
- If students need help the teacher will try to be explored the question's answer by asking questions to the student

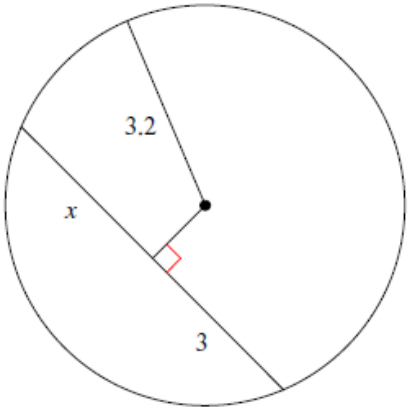
Worksheet

Power of a point with respect to circles

1. Find x.

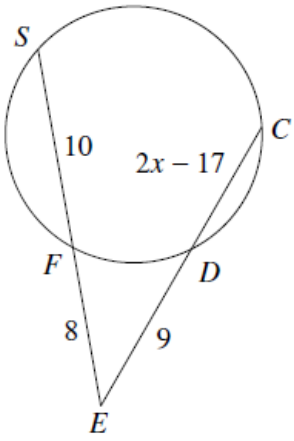


2. Find x.



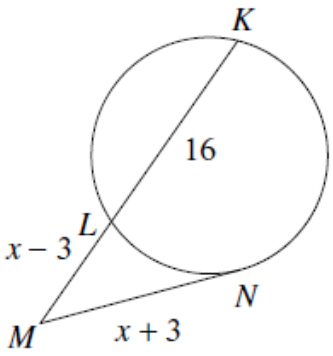
3.

Find *CE*



4.

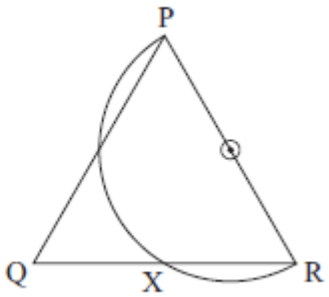
Find *NM*



5.

Triangle PQR is isosceles with PQ = PR.
A semi-circle with diameter [PR] is drawn and it cuts [QR] at X.

Prove that X is the midpoint of [QR] .

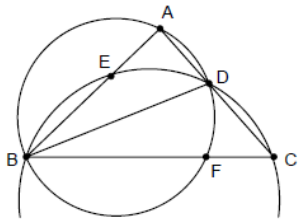


Transition: Now, I will distribute a sheet which has only one question and I want you to accomplish it in five minutes. After 5 minutes I will take them again.

E. Evaluate (5 minutes)

- The teacher will distribute a one question sheet and collect them.

Let BD be the angle bisector of angle B in triangle ABC with D on side AC. The circumcircle of triangle BDC meets AB at E, while the circumcircle of triangle ABD meets BC at F. Prove that $AE = CF$.



Solution

By intersecting chords theorem one has $AE \times AB = AD \times AC$, or

(1) $$AE = \frac{AD \times AC}{AB}.$$

Similarly, $CF \times CB = CD \times CA$ and therefore

(2) $$CF = \frac{CD \times CA}{CB}.$$

Dividing (1) by (2) gives

$$\frac{AE}{CF} = \frac{AD \times CB}{AB \times CD} = 1,$$

the last equality holds since $\frac{AD}{CD} = \frac{AB}{CB}$ by angle bisector theorem.

- Observe the students during the lesson and evaluate each student's responses to oral questions.
- Observe students as they apply new concepts

9. Closure & Relevance for Future Learning

- The teacher will want students to summarize what they learned during the lesson.
- The teacher will want students to write what specifically learned today at the back of their name cards and she will take them.

- Ask students are there any points not understood or any question.
- Then say them ”Ok, thank you so much for this enjoyable lesson”

10. Specific Key Questions:
    - “What positions can be a point of X ? ” (Analysis)
    - “What happens to the P(x) function when the point of X is on the circle?” (Analysis)
    - “Why you find 0 when the X is on the circle?” (Analysis)
    - “Can it be true for all points on circle?” (Comprehension)
    - “ What did you realize about when the point is in the exterior region of the circle?” (Analysis)
    - “ What did you realize about when the point is in the interior region of the circle?” (Analysis)

11. Modifications
    - If students cannot remember previous lesson, give them some clues.
    - If students do not give answer to your questions, wait 20 seconds more.
    - If the students cannot share any idea the teacher will wait twenty seconds more.
    - If the students need more time while they solve the exercises the teacher will give one more minutes.
    - If the students cannot solve the exercises the teacher will come back the definitions or previous examples and explain again
    - If students confuse about the answer clarify it by another way.
    - If they cannot solve the problems or do not understand, want another student to explain their friends how they used which method