Date: March 27, 2014
Teacher: Tuğba Özcan
Number of Students: 22
Grade Level: 10
Time Frame: 70 minutes

Period in Trigonometric Functions

1. Goal(s)
- Students will be able to determine the period and amplitude of trigonometric functions.

2A. Specific Objectives (measurable)
- Students will be able to find the period of trigonometric functions
- Students will be able to apply periodic functions to describe the location of a point in the plane by polar coordinates
- Students will be able to evaluate their learning skills and understanding by word problems and real life examples

2B. Ministry of National Education (MoNE) Objectives
- Periyodu ve periyodik fonksiyonu açıklar, trigonometrik fonksiyonların periyotlarını bulur.
- Trigonometrik fonksiyonların grafiklerini çizer.

2C. NCTM-CCSS-IB or IGCSE Standards:
- Students should use trigonometric relationships to determine lengths and angle measures (NCTM).

3. Rationale
- Students will use periodic functions throughout calculus so they will need to know their characteristics. One of these functions is to describe the location of a point in the plane by polar coordinates, an alternative to rectangular coordinates.
- Students should use periodic trigonometric functions to solve problems in, and gain insights into, other disciplines such as physics and other geometry topics and other areas of interest such as geology, astronomy, medicine, geophysics, statistics, and meteorology, and in music.
- Students will be able to see several examples of periodic trigonometric functions in real life. For example, spring harmonic motion or sound waves in physics, electrocardiograms in medicine, and sound frequency in music.

4. Materials

- Board markers
- Computer and Projector
- Worksheets, exploration sheets (there are 22students in class and copy each of them for 22 students)

5. Resources

- Hease and Herris Mathematics for the International Students SL and HL
- Hease abd Herris Cambridge International Mathematics

6. Getting Ready for the Lesson (Preparation Information)

- Copy activity sheets for each student. There are 22 students in the class.
- Before the lesson, open the projector.
- Monitor students while they are working and help them if they need.

7. Prior Background Knowledge (Prerequisite Skills)

- Students will be expected to know the basic properties of the unit circle
- Students will be expected to familiar with what the trigonometric functions are and how they are drawn in coordinate system
- Students will be expected to state and apply trigonometric functions

Lesson Procedures

Transition: Hello Guys! I am coming from Bilkent University and I will be teaching this lesson.

8A. Engage (5 minutes)

- Show the pictures by using projector. The pictures are related to the periodic situations in daily life

- Ask students first "What do you think how your heart beats? " and "Look at the picture, please. What can be the lesson about? "
- Give the possible responses from students and ask "What does the periodic mean?" and then ask them "What kind of periodic situations occur in the physical world?"
- Wait for a second for students' responses and discuss their answers.
- Next give possible examples of periodic phenomena other than students' answers.
- For example, seasonal variations in our climate, variations in average maximum and minimum monthly temperatures, the number of daylight hours at a particular locations, the phases of the moon or animal population
- Then tell the students " These phenomena illustrate variable behavior which is repeated over time . The repetition may be called periodic, oscillatory, or cyclic in different situations. Today we will see how trigonometric functions can be used to model periodic phenomena.

Transition: Let's discover how we can find period of trigonometric functions

B. Explore (15 minutes)

- Distribute the investigation sheet below. Then open the mathematica program and show first sinx, sin2x, 2sinx and the other functions on the table below. And want the students to complete the table by looking at the graphs of the functions on mathematica.

- Then ask the students " In which points does the function have the same value? " and "What is the max and min values of functions?" ," What is the difference between the max and min value of functions?" for every function.
- And ask the students what the rule or pattern is. Wait some seconds for students think about and then discuss their responses.
- Respectively show the cos and tan functions on the tables below. And similarly ask the same questions as above for every function.
- After the students complete the tables below ask students how the period and amplitude of the functions chance when the value of the coefficient of x change. What is the relationship between them?

Investigation Sheet

Period and amplitude of trigonometric functions

Complete the tables below and state how a and b affect the functions y= asinx, y= sinbx, y= acosx, y= cosbx, , y= atanx, y= tanbx, respectively.

a	Function	Maximum	Minimum	Period	Amplitude
1	y= sinx	1	-1	2π	1
2	y= 2sinx				
0.5	y= 0.5sinx				
-1	y= -sinx				
a	y= asinx				

b	Function	Maximum	Minimum	Period	Amplitude
1	y= sinx	1	-1	2π	1
2	y= sin2x				
0.5	y= sin0.5x				
-1	y= sin(-x)				
b	y= sin(bx)				

a	Function	Maximum	Minimum	Period	Amplitude
1	y= cosx	1	-1	2π	1
2	y=2cosx				
0.5	y= 0.5cosx				
-1	y= -cosx				
a	y= acosx				

b	Function	Maximum	Minimum	Period	Amplitude
1	y= cosx	1	-1	2π	1
2	y= cos2x				
0.5	y= cos0.5x				
-1	y= cos(-x)				
b	y= cos(bx)				

a	Function	Maximum	Minimum	Period	Amplitude
1	y= tanx	1	-1	2π	1
2	y=2tanx				
0.5	y= 0.5tanx				
-1	y= -tanx				
a	y= atanx				

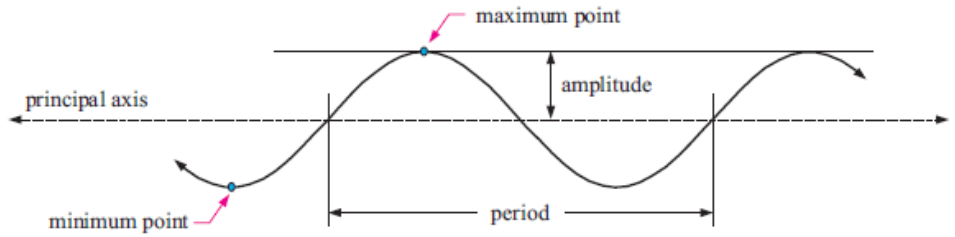
b	Function	Maximum	Minimum	Period	Amplitude
1	y= tanx	1	-1	2π	1
2	y= tan2x				
0.5	y= tan0.5x				
-1	y= tan(-x)				
b	y= tan(bx)				

Transition: Since you have already discovered the rule let's explain mathematical meaning.

C. Explain (10 minutes)

- Reflect the explanation below by using projector and explain.

TERMINOLOGY



- A **periodic function** is one which repeats itself over and over in a horizontal direction.
- The **period** of a periodic function is the length of one repetition or cycle.
- The graph oscillates about a horizontal line called the **principal axis** or **mean line**.
- A **maximum point** occurs at the top of a crest.
- A **minimum point** occurs at the bottom of a trough.
- The **amplitude** is the vertical distance between a maximum or minimum point and the principal axis.

You should have observed that for $y = a\sin(bx)$ and $y = a\cos(bx)$:

- a affects the **amplitude** of the graph. It provides a stretch with invariant x-axis and scale factor a.
- b affects the **period** of the graph. It provides a stretch with invariant y-axis and scale factor b.
- the amplitude is $|a|$ and the period is $\frac{360^o}{b}$.

Tangent Function: $y = \tan(x)$

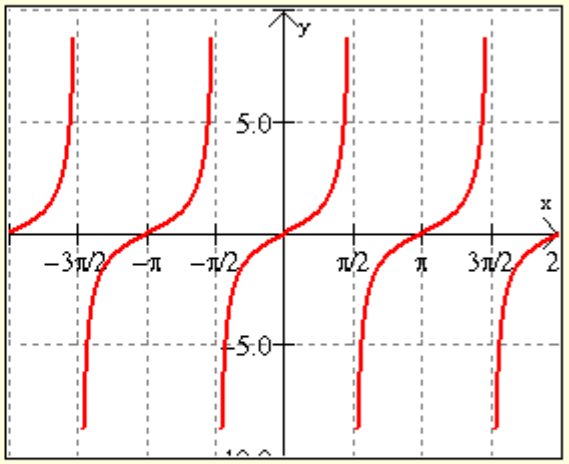
- One cycle occurs between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
- There are vertical asymptotes at each end of the cycle.

The asymptote that occurs at $\frac{\pi}{2}$ repeats every π units.

- period: π
- amplitude: none, graphs go on forever in vertical directions.

Note: a graphing utility, such as the one used to produce these graphs, may not show the function approaching infinity (going on forever upward or downward). The graphs, however, DO tend toward positive and negative infinity and do not STOP.

$y = \tan(x)$



The graph does not STOP even though the plot may "appear" as if the graph stops as the y-values increase/decrease.

- Give the example below

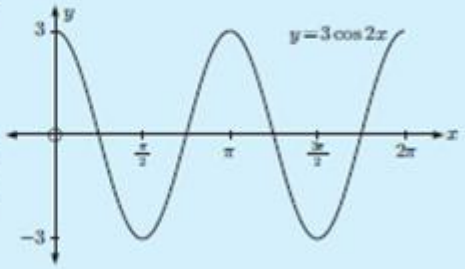
Example 5 — Self Tutor

Without using technology, sketch the graph of $y = 3\cos 2x$ for $0 \leqslant x \leqslant 2\pi$.

Notice that $a = 3$, so the amplitude is $|3| = 3$.

$b = 2$, so the period is $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$.

To obtain this from $y = \cos x$, we have a vertical stretch with scale factor 3 followed by a horizontal stretch with scale factor $\frac{1}{2}$, as the period has been halved.



Transition: I am distributing the worksheet, make a group of four and solve these problems

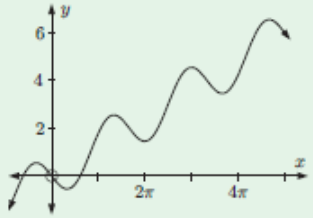
D. Extend (35 minutes)

- Distribute key paper and the worksheets and give clear instruction
- Explain the rubric of the worksheet and want the students to perform in a group according to the rubric
- Use the rubric to evaluate students work

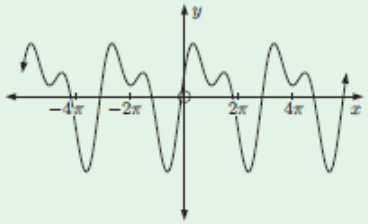
Worksheet

1 Which of the following graphs display periodic behaviour?

a



b



2 Draw the graph of $y = 4\sin x$ for $0 \leqslant x \leqslant 2\pi$.

3 State the minimum and maximum values of:

a $1 + \sin x$ **b** $-2\cos 3x$

4 State the period of:

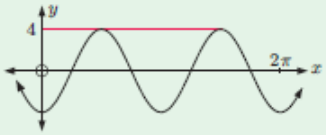
a $y = 4\sin(\frac{x}{5})$ **b** $y = -2\cos(4x)$ **c** $y = 4\cos(\frac{x}{2}) + 4$ **d** $y = \frac{1}{2}\tan(3x)$

5 Complete the table:

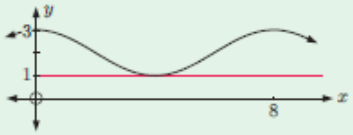
Function	*Period*	*Amplitude*	*Domain*	*Range*
$y = -3\sin(\frac{\pi}{4}) + 1$				
$y = \tan 2x$				
$y = 3\cos \pi x$				

6 Find the cosine function represented in each of the following graphs:

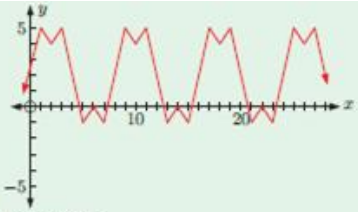
a



b



1 Consider the graph alongside.



a Explain why this graph shows periodic behaviour.

b State:

i the period

ii the maximum value

iii the minimum value

2 Find b given that the function $y = \sin bx$, $b > 0$ has period:

a 6π **b** $\frac{\pi}{12}$ **c** 9

3 **a** Without using technology, draw the graph of $f(x) = \sin(x - \frac{\pi}{3}) + 2$ for $0 \leqslant x \leqslant 2\pi$.

b For what values of k will $f(x) = k$ have solutions?

4 On the same set of axes, for the domain $0 \leqslant x \leqslant 2\pi$, sketch:

a $y = \cos x$ and $y = \cos x - 3$ **b** $y = \cos x$ and $y = \cos(x - \frac{\pi}{4})$

c $y = \cos x$ and $y = 3\cos 2x$ **d** $y = \cos x$ and $y = 2\cos(x - \frac{\pi}{3}) + 3$

5 The table below gives the mean monthly maximum temperature for Perth Airport in Western Australia.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp (°C)	31.5	31.8	29.5	25.4	21.5	18.8	17.7	18.3	20.1	22.4	25.5	28.8

a A sine function of the form $T \approx a\sin(b(t - c)) + d$ is used to model the data. Find good estimates of the constants a, b, c, and d without using technology. Use Jan $\equiv$ 1, Feb $\equiv$ 2, and so on.

b Check your answer to **a** using technology. How well does your model fit?

6 State the transformation(s) which map(s):

a $y = \cos x$ onto $y = \cos(x - \frac{\pi}{3}) + 1$ **b** $y = \tan x$ onto $y = -2\tan x$

c $y = \sin x$ onto $y = \sin(3x)$

Transition: I am evaluating your performance in group work according to the rubric

E. Evaluate (during the whole class and according to the rubric) (rubric attached at the end of the lesson plan)

- Assesses students' knowledge and skills through oral questions.
- Evaluate each students performance in group work by using the rubric while they are studying the worksheet
- Observe the students during the lesson and check each student's answer.
- Want the students to write a short paragraph about what they learn today and where they can use this information in their daily life

9. Closure & Relevance for Future Learning (5 minutes)

- Ask students are there any points not understood.
- Ask the students "What did you learn today?"
- Then say them "Ok, thank you so much for this enjoyable lesson"

10. Specific Key Questions:

- Does the function y= cosx repeat itself? (knowledge)
- If it repeats itself , in with points it has the same values? Explain all different values which repeat. (application)
- What does the periodic mean?(knowledge)
- What kind of periodic situations occur in the physical world? (evaluation)
- What did you notice in this problem?(analysis)
- How can we generalize the way that we observed?(synthesis)
- What if we chance the function then do the period be the same?(application)
- Why the such important period in trigonometric functions for us?(application)

11. Modifications

- If students cannot remember previous lesson, give them some clues.
- If students do not give answer to your questions, wait 20 seconds more.
- If the students cannot solve the exercises the teacher will come back the definitions or previous examples and explain again
- If students confuse about the answer clarify it by another way
- If they cannot solve the problems or did not understand, want another student to explain their friends how they used which method

Scoring Rubric-Cooperative Group Work
Criteria: Excellent/Good/ Needs Improvement

E	G	NI	Contributions:	Comments:
			Provides useful ideas. A definite leader who contributes a lot of effort.	
			Consistently stays focused on the task and what needs to be done. Very self directed.	
			Almost always listen to, shares with, and supports the efforts of others.	
			Keeps a positive attitude about the task.	