Date: March 6, 2014
Teacher: Tuğba Özcan
Number of Students: 15
Grade Level: 9
Time Frame: 45 minutes

Median of a Triangle

1. Goal(s)

- Students will understand the concept of median of a triangle and center of gravity
- Students will know about the properties of the median of a triangle.

2A. Specific Objectives (measurable)

- Students will be able to apply the definitions of the median of a triangle.
- Students will be expected to use geometric concepts and properties with spatial reasoning to solve problems in fields such as engineering.
- Students will be expected to use the median of a triangle to develop the center of gravity

2B. Ministry of National Education (MoNE) Objectives

- Üçgenin kenarortaylarının bir noktada kesiştiğini gösterir ve kenarortayla ilgili özellikleri açıklar.
- Kenarortayların kesiştiği noktanın üçgenin ağırlık merkezi olduğu vurgulanır; üçgenin ağırlık merkeziyle ilgili özellikler incelenir.
- Cetvel-pergel veya dinamik geometri yazılımlarında bunların karşılığı kullanılır.

2C. NCTM-CCSS-IB or IGCSE Standards:

- Students should establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others (NCTM).
- Students should use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (NCTM).
- Students should prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point (CCSS).

3. Rationale
   - Students will need to know the median of a triangle to use this idea in solving problems in other mathematics topics
   - Students should use the principles of the median of a triangle to solve problems in, and gain insights into, other disciplines such as physics and other geometry topics and other areas of interest such as art and architecture.
   - Students will learn about the properties of median and its relations with the center of gravity and how they are applicable to everyday life.

4. Materials
   - Board markers
   - Computer
   - Projector
   - Worksheets, activity paper, proof paper, short answer test (there are 15 students in class and copy each of them for 15 students)

5. Resources
   - TED 9th grade geometry book
   - McDougal Littell Geometry Book
   - PhSchool Teacher resources www.phschool.com/math

6. Getting Ready for the Lesson (Preparation Information)
   - Copy activity sheets for each student. There are 15 students in the class.
   - Before writing open the projector and show the activity sheet on the boardexplain students the instructions given in the activity sheet
   - Monitor students while they are working and help them if they need
   - Write students findings on the board

7. Prior Background Knowledge (Prerequisite Skills)
   - Students will be expected to know the basic properties of a triangle
   - Students will be expected to know to draw vertex in triangle
   - Students will be expected to familiar with the bisector of a triangle

Lesson Procedures

Transition: Good morning everyone. Today, we're going to study the median of a triangle.

8A. Engage (5 minutes)

- Show the video of "center of gravity" for 2 minutes http://www.youtube.com/watch?v=ajTyhbvMEAg
- Then ask questions students ;"What do you think about center of gravity?", "Why the center of gravity such important for us?"
- Ask "How is the center of gravity related with the median of a triangle?"

Transition: I am distributing this activity sheet and before starting I want you to listen to me, please.

B. Explore (5 minutes)

- Distribute the activity sheet(attached in this lesson plan)
- Give a clear instruction of the activity sheet
- Wait 10 minutes for the student to solve the questions in the worksheet

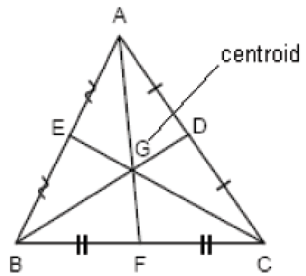
Transition: let's talk about what you find

C. Explain (20 minutes)

- Explain the law of median of a triangle. You can use the definition below

Median of a triangle is a line segment joining a vertex to the midpoint of the opposing side.

The medians are concurrent. Their intersection point is the centroid (center of mass) of the triangle.



Constructing the median of a given triangle:

Given: Triangle ABC
Construct: The median of the side [BC].
Procedure:

1. Begin with triangle PQR.
2. With the compass point on vertex B, set the compass width to any medium setting.
3. Draw an arc on each side of the line BC.
4. Without changing the compass width, place the compass point on vertex C, and make two more arcs so they intersect with the first two.
5. Draw a line between the points where the arcs cross. This will bisect the triangle side, dividing it into two equal parts. Label this point F.
6. Draw a line between F and A-the vertex opposite.
7. Therefore AF is the median of the side [BC].

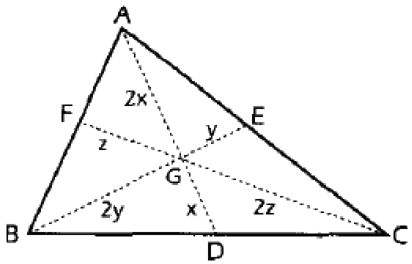
The other two medians can be constructed in a similar way

- Encourages students to explain concepts and definitions in their own words
- Asks for justification and clarification from students

- Give the first property of the median which is

Properties:

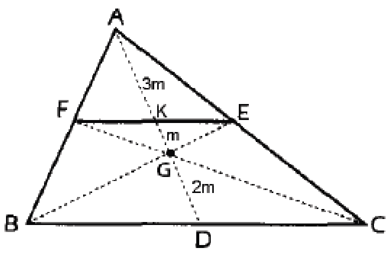
1)



$$|GD| = \frac{1}{3}|AD|,$$
$$|AG| = \frac{2}{3}|AD|$$

- Get the students solve the 5 examples of this property (Examples are chosen from TED workbook) (worksheet 1). But for first question give the students 1 minute to get them try to solve the question then make the students explain how they solve it by oral questions and then explain clearly
- Similarly, explain the other four questions
- Next, pass the second property of the median of a triangle

2)



- $[FE] \parallel [BC]$
- $|FE| = \frac{|BC|}{2}$
- $|AK| = |KD|$
- $|KG| = \frac{|AD|}{6}$

- Solve 5 questions by like the first property questions on worksheet 2 (Examples are chosen from TED workbook)
- Get the students solve the 5 examples of this property (Examples are chosen from TED workbook). But for first question give the students 1 minute to get them try to solve the question then make the students explain how they solve it by oral questions and then explain clearly
- Similarly, explain the other four questions in this manner

Transition: I am distributing the proof paper please try to tour best

D. Extend (10 minutes)

- Distribute the proof paper and give clear instruction
- Walk around and ask “How did you get this conclusion?”
- Check the students whether they solve the problems or not
- Asks for justification and clarification from students
- Encourages students to explain concepts and definitions in their own words

Transition: let's complete the sentences of the paper

E. Evaluate (5 minutes)

- Assesses students’ knowledge and skills through oral questions.
- Observe the students during the lesson and check each student’s answer.
- Distribute the True/False and short answer test (attached in lesson plan)

9. Closure & Relevance for Future Learning

- Ask students is there any points not understood.
- Then say them”Ok, thank you so much for this enjoyable lesson”
- Give the students their homework
- State the next topic of the lesson

10. Specific Key Questions:

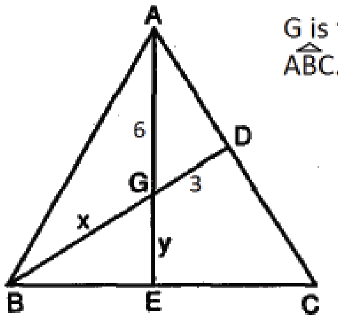
- How can you define the median of a triangle?(knowledge)
- What did you notice in this problem?(analysis)
- How can we generalize the way that we observed?(synthesis)
- What if we chance the triangles then what would the median be?(application)
- What do you think about center of gravity?(comprehension)
- Why the center of gravity such important for us?(application)
- How is the center of gravity related with the median of a triangle?(synthesis)

11. Modifications

- If students cannot remember previous lesson, give them some clues.
- If students do not give answer to your questions, wait 20 seconds more.
- If the students cannot share any idea the teacher will wait twenty seconds more.
- If the students need more time while they solve the exercises the teacher will give one more minutes.
- If the students cannot solve the exercises the teacher will come back the definitions or previous examples and explain again
- If students confuse about the answer clarify it by another way
- If they cannot solve the problems or did not understand, want another student to explain their friends how they used which method

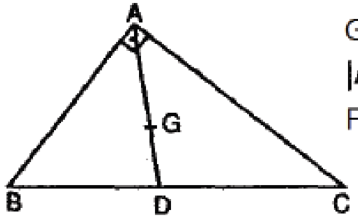
WORKSHEET 1

Ex(113):



G is the centroid of $\widehat{ABC}$. Find x+y.

Ex(114):

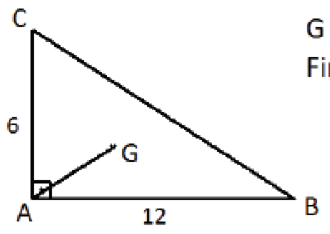


G is the centroid of $\widehat{ABC}$.

$|AG| = 16$ cm

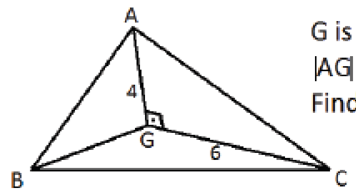
Find $|BC|$.

Ex(115):



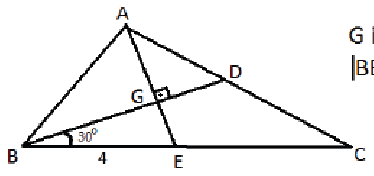
G is the centroid of $\widehat{ABC}$.
Find $|AG|$.

Ex(116):



G is the centroid of $\widehat{ABC}$.
$|AG|=4cm$, $|CG|=6cm$ are given.
Find $|BG|$.

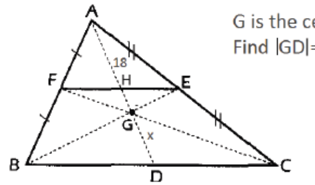
Ex(117):



G is the centroid of $\widehat{ABC}$.
$|BE|=4cm$. Find $|AC|$.

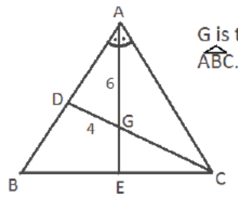
WORKSHEET 2

Ex(118):



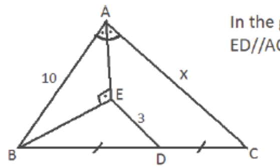
G is the centroid of $\widehat{ABC}$.
Find $|GD|=x$.

Ex(119):



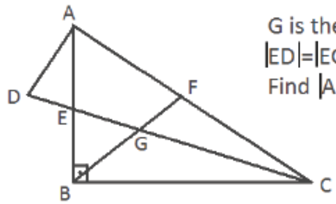
G is the centroid of $\widehat{ABC}$. Find $|AC|$.

Ex(120):



In the given figure, ED//AC. Find x.

Ex(121):



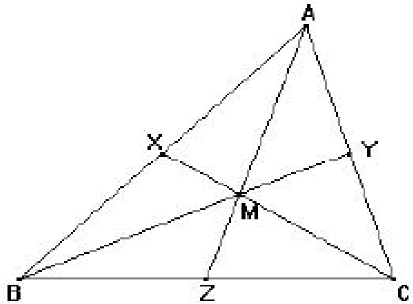
G is the centroid of $\widehat{ABC}$.
$|ED|=|EG|$, $|AD|=6cm$.
Find $|AC|$.

ACTIVITY SHEET (for exploration part)

Finding the Center of Mass of a Triangle

Definition: The point *M* where the medians of a triangle meet is called the **center of mass** or **centroid** of the triangle.

1. Construct triangle ΔABC (use triangle tool)
2. Find the midpoint *X*, *Y*, and *Z* of sides $\overline{AC}$, $\overline{AB}$, and $\overline{BC}$ (use midpoint tool)
3. Construct the medians, that is, the segments $\overline{AZ}$, $\overline{BY}$, and $\overline{CX}$. (use segment tool)



4. Drag a vertex, *A*, *B*, or *C*. Is there ever a time when the medians $\overline{AZ}$, $\overline{BY}$, and $\overline{CX}$ do not intersect?
   No, the lines are always intersecting each other.
5. Do the midpoints *X*, *Y*, and *Z* remain midpoints when the vertices of the triangle are dragged?
   Yes.
6. What do you notice about point *M* when dragging any of the three vertices?
   It moves with the vertices.
7. Create segments from $\overline{BM}$, $\overline{ZM}$, $\overline{CM}$, $\overline{YM}$, $\overline{XM}$, and $\overline{AM}$. (use segment tool)
8. Measure each new segment. (use measure tool)
9. Calculate *BM*/*BY*, *CM*/*CX*, and *AM*/*AZ*. (use calculate tool)

What are the results?

BM/*BY* = 0.67 *CM*/*CX* = 0.67 *AM*/*AZ* = 0.67

10. What do you notice about these results?

The ratio from the vertex to the midpoint and the vertex to the median point of the opposite side for each of the lines is exactly the same.

Short Answer Test

Complete the sentences and answer the "why" question.

1.In any triangle there can only be ....median(s). Why?

2.In an equilateral triangle all the medians are of .... length. Why?

3. In an isosceles triangle, the two medians drawn from the vertices of the equal angles are ....in length. Why?

4.In a scalene all the medians are of ....length. Why?

5.The medians are always ....the triangle. Why?

6. The median of a triangle divides the triangle into two triangles with ....areas. Why?

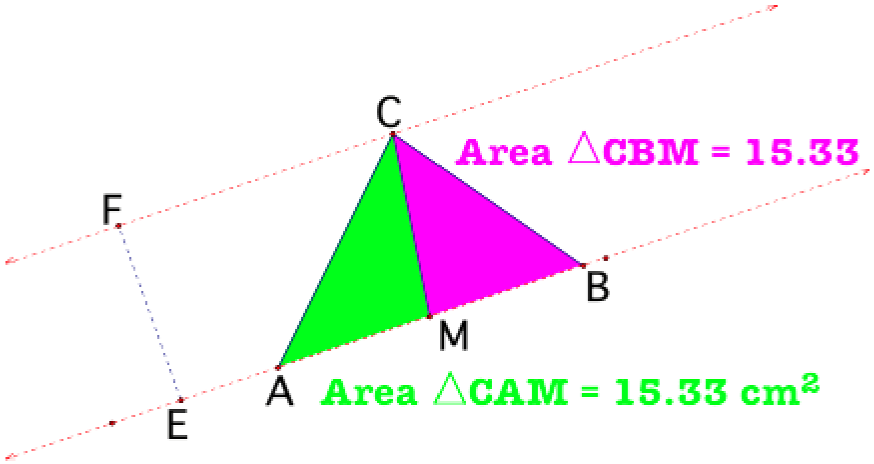
ANSWER SHEET

1. three
2. same
3. equal
4. different
5. inside
6. equal

Proof Paper

From the diagram in figure 1 we see that the two triangles CMA and BCM are equal in area. We can come up with a conjecture and say that, the median of a triangle divides the triangle into two triangles with equal areas.

To show that this is always true can you write a short proof?



The Proof

To show that this is always true we can write a short proof:
Area of any triangle = half the base x height.
In the triangles CMA and CBM, AM and MB are the bases respectively. The two triangles have the same height.
Therefore area of triangle CMA= ½(AM)(FE)
And area of triangle CBM=1/2(MB) (FE)
From the two areas we see that FE=FE (the two triangles have the same height).
Also AM=MB (M is the midpoint of AB, since AM is the median of the triangle. This then means that the two triangles are equal in area.

Now let us consider two medians. Look at the diagram below. We want to see if we can say anything else about the areas.
We have already seen that the median divides the triangle in two equal areas.
Let us extend that and see what happens when we put in the second median.
Look at the GSP diagram in figure 2

Two medians