Date: May 15, 2014
Teacher: Tuğba Özcan
Number of Students: 18
Grade Level: 11 IB-SL
Time Frame: 45 minutes

MATRICES

1. Goal(s)

- Students will be able to develop an understanding about the concept of matrices

2A. Specific Objectives (measurable)

- Students will be able to define a matrix, determine its dimension and find the intended row, column and element.
- Students will be able to differentiate a row matrix, a column matrix, a zero matrix, a square matrix, a diagonal matrix, a triangular matrix and express the identity matrix.
- Students will be able to add two matrices and identify the properties of addition of matrices.
- Students will be able to find the transpose of a matrix and identify its properties.
- Students will be able to multiply a matrix by a scalar and identify its properties.

2B. Ministry of National Education (MoNE) Objectives

- Matrisi örneklerle açıklar, verilen bir matrisin türünü belirtir ve istenilen satırı, sütunu ve elemanı gösterir.
- Kare matrisi, sıfır matrisini, birim matrisi, köşegen matrisi, alt üçgen matrisi ve üst üçgen matrisi açıklar, iki matrisin eşitliğini ifade eder.
- Matrislerde toplama işlemini yapar, bir matrisin toplama işlemine göre tersini belirtir, toplama işleminin özelliklerini gösterir ve iki matrisin farkını bulur.
- Bir matrisi bir gerçek sayı ile çarpma işlemini yapar ve özelliklerini gösterir.
- Bir matrisin devriğini (transpozunu) bulur ve özelliklerini gösterir.

2C. NCTM-CCSS-IB or IGCSE Standards:

- Students should understand matrices as systems that have some of the properties of the real-number system (NCTM).
- Students should develop an understanding of properties of, and representations for, the addition of matrices (NCTM).
- Students should develop fluency in operations with matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases (NCTM)

3. Rationale

- Students should recognize that the topic of matrices is useful for organizing data, and solving systems of linear equations. In fact, it is crucial for the systematic study of discrete problems in all branches, in particular computer science or industrial engineering. Especially for gamers with fast moving 3D graphics, the graphics card (the GPU) in a computer does nothing but billions of matrix calculations per second to render 3D objects

nicely on a 2-dimensional surface (the screen).

- Students will need to know the matrix topic because of the national exam (in this topic there are 3 questions in 2013 exam)

4. Materials
    - Colored board markers
    - Computer and Projector
    - PowerPoint presentation including the definitions of specific matrices and properties of matrix operations**.**
    - Worksheets (there are 18 students in class and copy each of them for 18 students)
5. Resources
    - TED College PowerPoint Presentation
    - MoNE Textbook for 11th grades.
    - http://www.mathsisfun.com/algebra/matrix-multiplying.html
    - http://www.shelovesmath.com/algebra/advanced-algebra/matrices-and-solving-systems-with-matrices/
6. Getting Ready for the Lesson (Preparation Information)
    - Prepare name cards for students and writes students' name on those cards before the lesson.
    - Copy worksheets for each student. There are 18 students in the class.
    - Prepare a checklist including student names.
    - Prepare a PowerPoint presentation including the definitions of specific matrices and properties of matrix operations**.**
    - Monitor students while they are working and help them if they need.
7. Prior Background Knowledge (Prerequisite Skills)
    - Students will be expected to know the basic algebraic operations
    - Students will be expected to familiar with the simultaneous linear equations

Lesson Procedures

Transition: Hello Guys! I am happy to meet you third time. I almost learned your names ; however I am going to distribute your name cards. Just put them on your desk. Thank you. And let's start our lesson.

8A. Engage (5 minutes)

- *T*he teacher tell the students "Today we are going to study the topic matrix. Do you have any idea about what the matrix is?" (Some possible answers will be related the film The Matrix.). At the same time teacher opens the PowerPoint and shows the first page.
- Then, teacher ask the students whether they know how to demonstrate/write a matrix.
- Then she writes $A=\begin{bmatrix}9 & 7\\ 6 & 8\end{bmatrix}$ and says “this is a matrix” and asks “why do you think we show numbers or variables in the array form? Why do you think it is useful?"
- Then, teacher shows the table below including these numbers and tells the students "we apply matrices for organizing the data for example."

Exam / Student	I. Exam	II. Exam	III. Exam
Zeynep	80	90	95
Utku	75	86	92
Tahsin	78	85	88

- Thus, the teacher emphasizes that matrices are useful for organizing data.
- The teacher gives extra information about the rationale of the topic.
- The teacher mentions about the Google’s page-rank algorithm which utilizes matrices of huge dimensions.
- Then the teacher opens the PowerPoint presentation and show the second and third pages.
- The teacher tells the students" Matrix is a way of representing data between two arrays. An $m\times n$ matrix has m rows and n columns, and each entry is given a unique name, based on its row and column: The matrix A is often denoted $[A]$."
- Next, teacher opens the 4th page of the presentation and tell the students "The dimension of a matrix is the number of rows and the number of columns in a matrix. For example, a 3×4 -dimensional matrix has 3 rows and 4 columns. Here is an example of a 3×2 -dimensional matrix. The number a_{12} is the number in the **1st row** and the **2nd column**. Thus, $a_{12}= 16$. a_{21} is the number in the **2nd row** and the **1st column**. Thus, $a_{21} = -12$."

$$A=\begin{bmatrix} 1 & 16 \\ -12 & -\frac{3}{2} \\ 4 & 0 \end{bmatrix}$$

Transition: Teacher distributes the Worksheet-1 and says "There are some special types of matrices that you will need to know while solving questions. Now, try to match these specific types of matrices with their names."

B. Explore (10 min.)

Worksheet 1
MATRICES

$$A = [a_{ij}]_{mxn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2n} \\ a_{31} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & \cdots & \cdots & \cdots & a_{mn} \end{bmatrix}_{mxn}$$

I. Match the specific types of matrices with their names.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix} \qquad C = \begin{bmatrix} 6 \\ 72 \\ 4 \\ 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 6 & 0 \\ 3 & 5 & 8 \end{bmatrix} \qquad F = \begin{bmatrix} 3 & 6 & 9 \\ 5 & -8 & 10 \\ 1 & 7 & 2 \end{bmatrix} \qquad G = [-1\ 3\ 5\ 2\ -3] \qquad H = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -2/7 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

1. row matrix ( )
2. column matrix ( )
3. zero matrix ( )
4. square matrix ( )
5. diagonal matrix ( )
6. identity matrix ( )
7. scalar matrix ( )
8. triangular matrix ( )

II. Fill in the blanks according to your matching above.

If a matrix is a row matrix…….
If a matrix is a column matrix……..
If a matrix is a zero matrix……..
If a matrix is a square matrix……...
If a matrix is a diagonal matrix…….
If a matrix is a identity matrix……..
If a matrix is a scalar matrix……..
If a matrix is a triangular matrix……

- Then teacher discusses the second part with the students.

Transition: "Now open your notebooks and take the notes of the definitions and properties on your notebook"

C. Explain (15 min.)

- Teacher opens the 16th page of the presentation and tells that
  Equivalent matrices: Two matrices A and B are equal if they have the same dimensions and corresponding elements are equal.
- (The teacher writes two matrices on the board and asks if the two matrices are equal or not.)

  * Are A and B equal? (Yes, they are)

$$A=\begin{bmatrix} 4 & \cos(0) \\ \sin(0) & -2 \end{bmatrix}$$

$$B=\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$$

- Teacher opens the 18th presentation and tells the students
  Addition of Matrices: Two matrices A and B can be added if they have the same dimension by adding corresponding elements. The result is also a matrix in the same dimension.

  * Find A+B

$$A=\begin{bmatrix} 10 & 13 \\ -11 & -8 \end{bmatrix}$$
$$B=\begin{bmatrix} -9 & -13 \\ 11 & 9 \end{bmatrix}$$

  * Find A+ B. (The teacher expects the students to realize that they cannot add two matrices which have different dimensions)

$$A=\begin{bmatrix} 10 & 13 \\ -11 & -8 \end{bmatrix}$$

$$B=\begin{bmatrix} 2 & 6 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$

- Teacher opens the 22nd page of the presentation and tells that

  Multiplication of a matrix by a scalar: If k $\in\mathbb{R}$ and A is any matrix, then kA is the new matrix for which each entry of A has been multiplied by k. The result is a matrix having the same dimension as A.

  * Find -½ A, where

$$A=\begin{bmatrix} 10 & 2 \\ -8 & 0 \end{bmatrix}$$

PROPERTIES

1) A+B=B+A (commutative property)
2) A+(B+C)=(A+B)+C (associative property)
3) A+0=0+A=A (identity of addition)
4) A+(-A)=(-A+A)=0 (additive inverse)
5) k(A+B)=kA+kB (distributive property)
6) (k + c) A= kA+cA (distributive property)
7) (k+A)=k.I+A, where k $\in \mathbb{R}$ and A is a square matrix.

- Teacher asks the students if we need to write the same set of rules for subtraction? WHY?

Transition: Teacher distributes the worksheet below. and wants students to work on the worksheet with their peers.

D. Extend (5 min.)

Worksheet 2
MATRICES

1) $A = \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix}$ are given. Find 2A+3B-C.

2) If $2X - 6\begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} = 2\begin{bmatrix} 1 & -1 \\ 0 & -3 \\ -1 & -5 \end{bmatrix}$, find the matrix X.

3) A and B are 2x2 matrices,
$4A-3B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ and $5A-6B = \begin{bmatrix} -7 & 1 \\ 6 & 0 \end{bmatrix}$ are given.
Find matrix A.

E. Evaluate (during the classroom discourse)

- Teacher evaluates students' performances during the exploration and extension processes by keeping a checklist.
- Observe the students during the lesson and assess them according to their responses to the questions asked throughout the classroom discourse.

9. Closure & Relevance for Future Learning (5 min.)
    - Ask students if there are any points not understood or any questions.
    - Teacher tells about the anecdote in the modification part.
    - Then say them "Ok, thank you so much for this enjoyable lesson"

10. Specific Key Questions:
    - Why do you think we show numbers or variables in an array form?
    - Why do you think it is useful?
    - Are these two matrices equal?
    - Can I add these two matrices?
    - Do we need to write the same set of rules for subtraction? Why?

11. Modifications

A matrix, very simply, is a rectangular array of numbers. Joseph James Sylvester developed matrices in 1851, and described many of their properties. As linear algebra was further developed, Charles Lutwidge Dodgson(AKA Lewis Carroll) objected to the term "matrix." He preferred the term "block," and argued this point in his book *Elementary Treatise on Determinants* -- his first book after *Alice in Wonderland*. Ironically, the fictional character Neo is told to "follow the white rabbit" in a movie that was **not** called "*The Block*".