MATHEMATICAL THINKING WITH PROBLEM SOLVING

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Abstract

This article was about how the problem solving should be implemented in the centre of mathematics classroom. According to George Polya (1945), problem solving was a process consisting of four steps. They were that first to understand the problem, second to create a plan, third to carry out plan, and finally, to look back and check the solution(as cited in Brahier,2010, p.26). In order to define more clearly, I gave an example problem and applied Polya's four steps to solve it. At the end, I realized how important the problem solving was in mathematics classroom and other areas of life.

Key Words: problem solving, mathematical process, Polya's four steps

Introduction

The purpose of this article was to respond to the question " What did it mean if someone suggested to place problem solving at the center in mathematics classroom?" With the help of my traditional experiences in teaching and learning mathematics, I exemplified problem solving. Moreover, I showed Polya's four step was so essential process that problem solving could be more understandable.

Problem Solving Process

To begin with, what did the problem solving mean? "Problem solving was a mathematical process by which students attempted to identify what was needed, to set up a plan, to implement the plan and check the reasonableness of their answer." (Polya,1945). That is, problem solving was just a process not about the answer.

Then, as The National Council of Teachers of Mathematics recommended, problem solving should have been the focal point of the mathematics curriculum (An Agenda for Action, 1980,para.1). Therefore, students could use problem solving in daily life and make connections between them. However, how should the problem solving be implemented in mathematics classroom? I had an answer to this question with respect to my traditional experiences. Which was that for a given problem, different from an exercise, I began to understand what was the problem and what was wanted me to find? Then, I identified the data given and utilized them. By the way, it was not forgotten that in order to solve a problem, first someone had an enough mathematics background. Hence, I solved the problem because of my prior mathematic knowledge. And finally, I substituted the answer into the problem and check if it was correct.

Similar to my own experiences, I learned in previous special teaching methods class that each person could state a different strategy in problem solving. However, George Polya's four steps were seen as the most fundamental approach in problem solving. With regard to Polya, to

solve a problem, someone should begin first to comprehend the problem. Known(s) and unknown(s) should be found. And with a piece of paper, decide how to solve it. A definition, an equation, or theorem could be used. Second, somebody should apply to similar problems or even simpler ones in the past. Also, all possibilities should be checked and impossible situations should be eliminated as well. Third, each step should be satisfied conditions and be provable. Finally, the result should be checked by substituting it into the statement.

Now, to express these steps more clearly, I wanted to give an example. The problem was that *" Everyone sits in a circle. People are numbered clockwise around the circle, starting with #1. Start with #1 and skip him or her. Person #2 is asked to stand up. Person #3 is skipped, and #4 is asked to stand up. This process continues around the circle. Skipping every other person, until only one remains seated. If there are 25 people, where should you sit to make sure you are the one remaining seated? Describe a rule or formula that will work for any number of people."* In first step, I tried to understand the problem. And I thought the numbers from 1 to 25 which aligned in a circle and supposed somebody erased these numbers with respect to a rule. That is, starting #1 and skipping it, one erased # 2. Then by skipping 3 and erasing 4, person could continue like that until there was just one number remained. For 25 numbers which number would remain? In second step, I could apply a simpler problem. i.e., thinking numbers 1 to 10 and with the same rule, I erased numbers and then I saw that # 5 remained. I observed the remaining number could not be an even number so I eliminated the impossible situations. In third, I thought whether or not it could be proven. Furthermore, I tried to find a general formula. Yet, I found there were loops and some relations with modulo and induction hypothesis in mathematics. By induction, I tried for numbers from 1 to 2. And, by supposing for any number n (from 1 to n) it was also true, I tried to prove for n+1. Meanwhile, by modular arithmetic, in the 1st loop (1,2; 1,2,3), the remaining numbers were 1 and 3, respectively.(Observe there were

2 cycle, i.e.,2=2 to the power 1). In 2nd (1,2,3,4; 1,2,3,4,5; ...;1,2,3,4,5,6,7) the remaining were 1,3,5,and 7, respectively.(4 cycle; 4=2 to the power 2) Continuing like this, I found, for the loop (1,2,3,4,5,6,7,8,..,16; ...1,2,3,4,...,25), that the remaining numbers were 1,3,5,7,9,11,13,15,17, and 19, respectively. Finally, in the last step, I looked back and check the statement if it was true any number.

Hence, problem solving process was a kind of strategy to find the solution. Besides, everyone could apply the same problem solving technique to solve other problems s/he faced in mathematics. Moreover, they could gain the analyzing ability in daily lives.

Conclusion

The purpose of this article was to put the problem solving into the centre of mathematics teaching and learning. Besides, to make the problem solving be applicable into the mathematics classroom. I explained this not only by the help of my own experiences in learning and teaching mathematics but also through Polya's four steps in problem solving. I learned that problem solving was a process unlike an answer. And, this process could help people to make analysis and synthesis other problems in mathematics and in casual life, as well.

References

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"A Bilkent student does not lie, cheat, or steal or tolerate those who do. On my honor, as a Bilkent student, I have neither given nor received unauthorized aid on this academic work."

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