

Date: 19.12.2013

Teachers: Tuğba Özcan,

Number of Students: 9

Grade Level: 10

Time Frame: 80 Minutes

GRAPHING QUADRATICS

1. Goal(s)

- To develop an understanding of graphs of quadratic functions.

2A. Specific Objectives (measurable)

- Students will be able to graph quadratic functions.
- Students will be able to find a quadratic from its graph.
- Students will also be able to determine whether a parabola opens upward or downward.
- Students will be able to identify the axis of symmetry.
- Students will be able to determine the equation of a parabola using the vertex form of a parabola.

2B. Ministry of National Education (MoNE) Objectives

- 10.6.2. İkinci Dereceden Fonksiyonlar ve Grafikleri,
- 10.6.2.1. İkinci dereceden bir değişkenli fonksiyon açıklar ve grafiğini çizer.

2C.IGCSE Standards:

- Recognition of quadratic functions from the shape of their graphs. (Functions – Extended curriculum 3.2)

3. Rationale

- Graphing of quadratics will help students to visualize quadratic equations
- Quadratic functions are widely used in science, business, and engineering. The U-shape of a parabola can describe the trajectories of water jets in a fountain and a bouncing ball.
- Students need all information so they can analyze the graphs and how they related to given situations in the problems and they will reach a conclusion about the nature of the roots

4. Materials

- Board.
- At least two different colored board markers.
- Worksheets
- Projector
- Computer
- A notebook in which there is a list of the names of the students and a pencil for assessment

5. Resources

- Haese&Harris Publications. Mathematics for the international student Mathematics HL (Core) book. The authors are Paul Urban, John Owen, David Martin, Robert Haese, Sandra Haese, and Mark Bruce.
- Haese&Harris Publications IGCSE Cambridge International Mathematics (0607) Extended book. The authors are Keith Black, Alison Ryan, Michael Haese, Robert Haese, Sandra Haese, and Mark Humphries.
- 10.sınıf Esen yayınları Matematik konu anlatımlı.

6. Getting Ready for the Lesson (Preparation Information)

- Before teaching, teacher will introduce herself.
- Teacher should make sure that she gets worksheets.
- Teacher will draw real life examples' graph on the board in the engagement part.
- Copy worksheets for each student before the class. There are 9 students in the class.
- Explain students the instruction given in the worksheet.
- Let the students study worksheets individually or with a group
- Monitor the students while they are studying, and help them if they need
- Each student will check her/ his pair's worksheet

7. Prior Background Knowledge (Prerequisite Skills)

- Students will be expected to have algebraic thinking skills
- Students will be expected to know linear equation and the difference between linear equation and quadratic equation
- Students will be expected to be familiar with the factorizing a quadratic equation

- Students should know what quadratics means.
- Student should know how to find the discriminant of a quadratic.
- Students should know to draw a linear equation.
- Students should know relationship between discriminant and roots of the quadratics.

Lesson Procedures

*Transition: **Part 1**-Hi guys! I am your mathematics teacher during one hour period. I am a trainee teacher from Bilkent University. I hope we will have a good lesson together. **Part 2**- So far, you have learned how to draw quadratics functions. If I show you a graph of a quadratic function, you think, can you find the equation in the quadratic form? Let's try.*

8A. Engage (5 minutes)

- Remind students that the topic of the lesson is the graphing a quadratic function
- Say students "You studied the quadratic functions last week and today we will see them in our daily life."
- Ask students" Do you know skate board ? What kind of motion do you observe there? and Imagine I have a ball which I obtained it from the papers that I did not use anymore in my hand and I want to throw it in the trash bin from at this point to there. What kind of motion do you observe this time?"
- Wait students to think about it and then discuss, let them answer the question.
- Want the students to open their notebook and sketch these graphs on it.

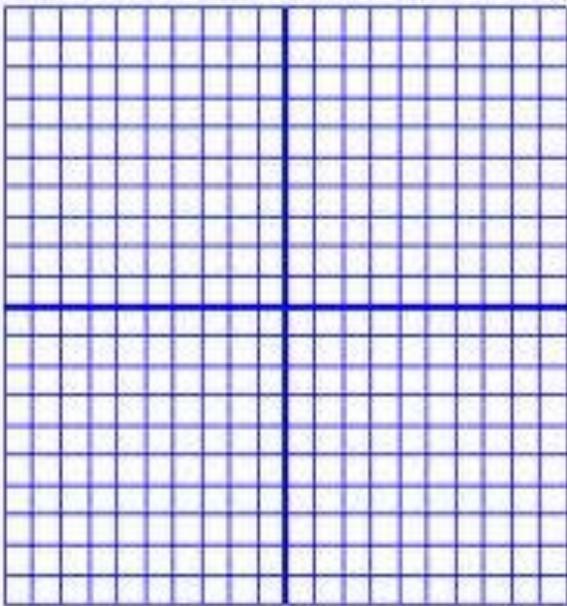
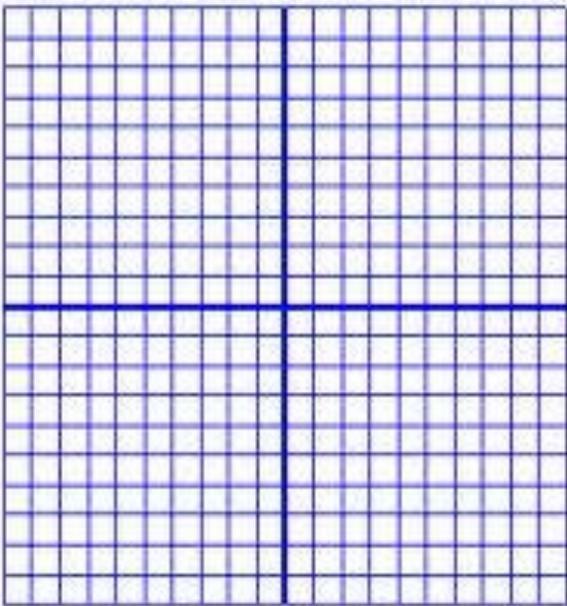
*Transition: **Part 1**- Students have already observed what kind of graphs can it be, say them "Now let's see what kind of graphs do the quadratic functions which are $y = x^2$ and $y = -x^2$ "*

B. Explore

Part 1-(10 minutes)

- Distribute the papers in which plotting dots are there (attached in this lesson plan)
- Give a clear instruction of the paper say "In this paper, you see it is a plot graph paper and I want you to give values to x that 0,1,2,3,4,5 and -1,-2,-3,-4,-5,..and so on in each quadratic function $y = x^2$ and $y = -x^2$. Then plot the points that you obtained for the y value."

PLOTTING GRAPH PAPER



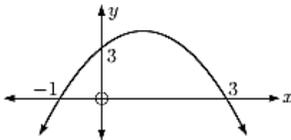
- Wait 10 minutes for the student to solve the questions in the paper
- By the way walk around the students and learn them whether or not they need to help by asking "Is there anything they are confused about?"
- Walk around the students and take notes in your notebook about their assessments by observing or asking them questions such as " Why do you think that?", " How did you reach that conclusion?"

Part 2- (20min.)

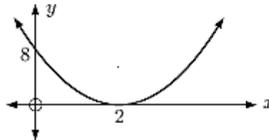
- Project the following g graphs one by one on the board, ask students to draw the graphs on their notebooks simultaneously.

Find the equation of the quadratic function with graph:

a



b

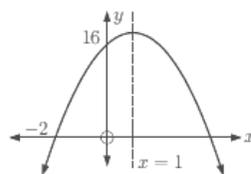


a Since the x -intercepts are -1 and 3 ,
 $y = a(x + 1)(x - 3)$.
 The graph is concave down, so $a < 0$.
 When $x = 0$, $y = 3$
 $\therefore 3 = a(1)(-3)$
 $\therefore a = -1$
 The quadratic function is
 $y = -(x + 1)(x - 3)$.

b The graph touches the x -axis at $x = 2$,
 so $y = a(x - 2)^2$.
 The graph is concave up, so $a > 0$.
 When $x = 0$, $y = 8$
 $\therefore 8 = a(-2)^2$
 $\therefore a = 2$
 The quadratic function is
 $y = 2(x - 2)^2$.

- Then, let them to try to find the quadratics functions.
- Give them 3minutes for each graph.
- Then collect the answers from the students and wrote the answers on the board.
- Make them realize their mistakes by asking "what do you think about your friends' answer? Why do you think it is correct or wrong?"
- Then draw other two graphs the following :

Find the equation of the quadratic function with graph:



The axis of symmetry $x = 1$ lies midway between the x -intercepts.

\therefore the other x -intercept is 4.

\therefore the quadratic has the form

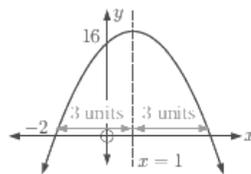
$$y = a(x + 2)(x - 4) \quad \text{where } a < 0$$

But when $x = 0$, $y = 16$

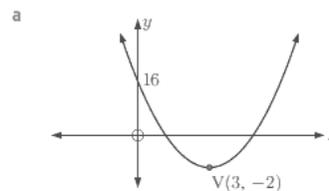
$$\therefore 16 = a(2)(-4)$$

$$\therefore a = -2$$

The quadratic is $y = -2(x + 2)(x - 4)$.

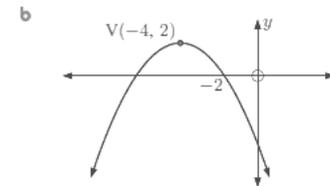


Find the equation of each quadratic function given its graph:



a Since the vertex is $(3, -2)$, the quadratic has the form
 $y = a(x - 3)^2 - 2$ where $a > 0$.
 When $x = 0$, $y = 16$
 $\therefore 16 = a(-3)^2 - 2$
 $\therefore 16 = 9a - 2$
 $\therefore 18 = 9a$
 $\therefore a = 2$

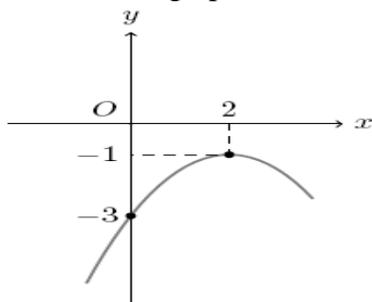
The quadratic is $y = 2(x - 3)^2 - 2$.



b Since the vertex is $(-4, 2)$, the quadratic has the form
 $y = a(x + 4)^2 + 2$ where $a < 0$.
 When $x = -2$, $y = 0$
 $\therefore 0 = a(2)^2 + 2$
 $\therefore 4a = -2$
 $\therefore a = -\frac{1}{2}$

The quadratic is
 $y = -\frac{1}{2}(x + 4)^2 + 2$.

- Ask students “ and now how can we find the quadratic forms of the graphs ?” and say “ please try to find them”
- Again, give the 4 minutes and collect the answers and check them with students.
- Give them last graph the following:



$$y = \frac{-x^2}{2} + 2x - 3$$

- Watch the same procedures as previous page for this graph.
- Finally, project the following question.

Find the equation of the quadratic whose graph cuts the x -axis at 4 and -3 , and which passes through the point $(2, -20)$. Give your answer in the form $y = ax^2 + bx + c$.

Since the x -intercepts are 4 and -3 , the quadratic has the form $y = a(x - 4)(x + 3)$ where $a \neq 0$.

When $x = 2$, $y = -20$

$$\therefore -20 = a(2 - 4)(2 + 3)$$

$$\therefore -20 = a(-2)(5)$$

$$\therefore a = 2$$

The quadratic is $y = 2(x - 4)(x + 3)$
 $= 2(x^2 - x - 12)$
 $= 2x^2 - 2x - 24$

- Give them two minutes and students will try to find the answer.

Transition: Part 1- Say students to stop working and ask them “What did you notice anything in these examples?” and wait for a second then after the discussion explain it. *Part 2-* So far, you all did a good job. Now make a brief summary.

C. Explain

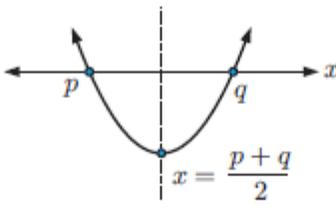
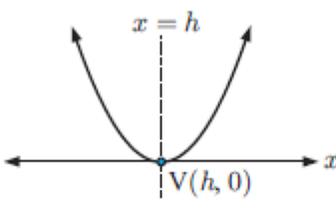
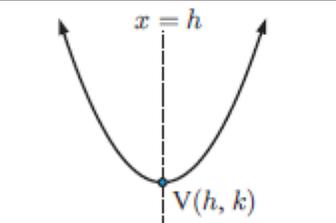
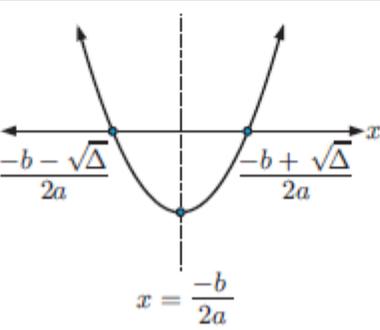
Part 1 (10 min)

- Then say students these graphs are called parabola. And explain how they are sketched and how the other quadratic functions are sketched. Use the notes below:

For a quadratic function $y = ax^2 + bx + c$, $a \neq 0$:

- $a > 0$ produces the shape  called concave up.
- $a < 0$ produces the shape  called concave down.
- If $-1 < a < 1$, $a \neq 0$ the graph is wider than $y = x^2$.
If $a < -1$ or $a > 1$ the graph is narrower than $y = x^2$.

Summary:

Quadratic form, $a \neq 0$	Graph	Facts
<ul style="list-style-type: none"> $y = a(x - p)(x - q)$ p, q are real 		<p>x-intercepts are p and q axis of symmetry is $x = \frac{p+q}{2}$ vertex is $(\frac{p+q}{2}, f(\frac{p+q}{2}))$</p>
<ul style="list-style-type: none"> $y = a(x - h)^2$ h is real 		<p>touches x-axis at h axis of symmetry is $x = h$ vertex is $(h, 0)$</p>
<ul style="list-style-type: none"> $y = a(x - h)^2 + k$ 		<p>axis of symmetry is $x = h$ vertex is (h, k)</p>
<ul style="list-style-type: none"> $y = ax^2 + bx + c$ 		<p>y-intercept c axis of symmetry is $x = \frac{-b}{2a}$ vertex is $(\frac{-b}{2a}, c - \frac{b^2}{4a})$ x-intercepts for $\Delta \geq 0$ are $\frac{-b \pm \sqrt{\Delta}}{2a}$ where $\Delta = b^2 - 4ac$</p>

- Solve the quadratic equations on the worksheet which you distributed in the exploration part on the board by questioning method
- Write a quadratic equation and ask students “what are the coefficients of the equation?” and “What are the roots that you found? And how many roots are there?” and “What is the discriminant and is it greater than or equal to zero or less than zero?”
- By the way be sure all the students have the same solution by asking students “Does anyone disagree with this answer?” and discuss
- Write the situations on the table.
- Say students to draw the table on their notebooks
- Wait for students to write

Part 2 (5 min)

- make a brief summary by using slideshow.
- If students have a problem in the explore part. Teacher will explain and solve the problem. One of the problems can be during the stating axis of symmetry.
- Before solving problems, teacher will ask all the class whether they have an idea about the questions or not.

*Transition: **Part 1-** What does this tell us? For what purpose we have already investigated the nature of the quadratic equations? And distribute the extension worksheet. **Part 2-** If you have no questions, let's solve more problems.*

D. Extend

Part 1(10 min):

- Explain the worksheet as “In this paper I want you to investigate
- Walk around the students and help them if they need or clarify the misunderstood points
- The worksheet attached below :

EXERCISE 1C.1

1 Using axes intercepts only, sketch the graphs of:

a $y = (x - 4)(x + 2)$

b $y = -(x - 4)(x + 2)$

c $y = 2(x + 3)(x + 5)$

d $y = -3x(x + 4)$

e $y = 2(x + 3)^2$

f $y = -\frac{1}{4}(x + 2)^2$

2 State the equation of the axis of symmetry for each graph in question 1.

3 Match each quadratic function with its corresponding graph.

a $y = 2(x - 1)(x - 4)$

b $y = -(x + 1)(x - 4)$

c $y = (x - 1)(x - 4)$

d $y = (x + 1)(x - 4)$

e $y = 2(x + 4)(x - 1)$

f $y = -3(x + 4)(x - 1)$

g $y = -(x - 1)(x - 4)$

h $y = -3(x - 1)(x - 4)$

The axis of symmetry
is midway between
the x -intercepts.



4 Use the vertex, axis of symmetry, and y -intercept to graph:

a $y = (x - 1)^2 + 3$

b $y = 2(x + 2)^2 + 1$

c $y = -2(x - 1)^2 - 3$

d $y = \frac{1}{2}(x - 3)^2 + 2$

e $y = -\frac{1}{3}(x - 1)^2 + 4$

f $y = -\frac{1}{10}(x + 2)^2 - 3$

5 Match each quadratic function with its corresponding graph:

a $y = -(x + 1)^2 + 3$

b $y = -2(x - 3)^2 + 2$

c $y = x^2 + 2$

d $y = -(x - 1)^2 + 1$

e $y = (x - 2)^2 - 2$

f $y = \frac{1}{3}(x + 3)^2 - 3$

g $y = -x^2$

h $y = -\frac{1}{2}(x - 1)^2 + 1$

i $y = 2(x + 2)^2 - 1$

- 1 Consider the quadratic function $y = \frac{1}{2}(x - 2)^2 - 4$.
 - a State the equation of the axis of symmetry.
 - b Find the coordinates of the vertex.
 - c Find the y -intercept.
 - d Sketch the function.
- 2 Solve the following equations:
 - a $x^2 - 5x - 3 = 0$
 - b $2x^2 - 7x - 3 = 0$
- 3 Solve the following using the quadratic formula:
 - a $x^2 - 7x + 3 = 0$
 - b $2x^2 - 5x + 4 = 0$

Part 2 (10min):

- Students will try to solve questions about graphing of quadratic equations on the worksheet.
- Walk around and ask “how did you get this answers?”
- Check the students whether they solve the problems or not.
- If they finish part 1, ask students to switch their worksheets with their pairs.
- Each student will check his/her pair’s answer.
- Then, answers will be checked on the board by writing the questions on the board.

Transition: Part 1- Part 2- Every mission has completed successfully.

E. Evaluate

Part 1(throughout the lesson):

- Assess students’ knowledge and skills through oral questions
- Observe each student during the lesson and take notes about their assessment while they are studying the worksheets
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Part 2 (During the whole lesson):

- Assesses students’ knowledge and skills through oral questions.
- Observe the students during the lesson.

- Take notes students' name if they have a problem when they solve questions.

9. Closure & Relevance for Future Learning

- Ask students to explain what they learn today.
- Then, want students to write information that they have learned this lesson on their notebooks.
- Assign students to do the rest of the questions on the worksheet.
- Ask students if there are any points not understood.
- Then say that "Ok, thank you so much for this enjoyable lesson"
- Give the students their homework
- State the next topic of the lesson

10. Specific Key Questions:

- How can you define the quadratic equations?(knowledge)
- How can you factorize a quadratic equation?(knowledge)
- What did you notice in this problem?(analysis)
- How can we generalize the way that we observed?(synthesis)
- Can you estimate what the pattern is?(evaluation)
- What if we change the coefficients of the quadratic then What would the discriminant be?(application)
- Can you explain why the mathematicians use this symbol or why they called the formula discriminant?(comprehension)

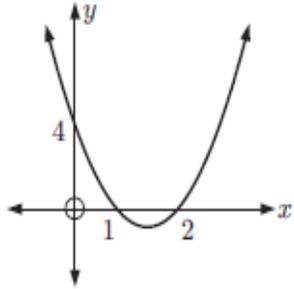
11. Modifications

- If students cannot remember previous lesson, give them some clues.
- If students do not give answer to your questions, wait 20 seconds more.
- Choose simple questions firstly to solve on the board.
- If the students cannot remember the factorization remind how to factorize
- If the students cannot give answer your questions wait 20second more or give them clues
- If students confuse about the answer clarify it by another way
- If they cannot solve the problems or did not understand, want another student to explain their friends how she or he did or which method they used

WORKSHEET

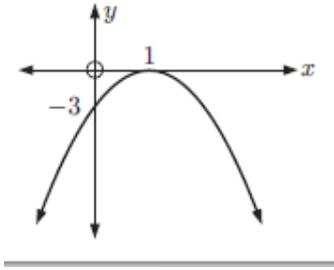
1. Find the equation of the quadratic with graph :

a-



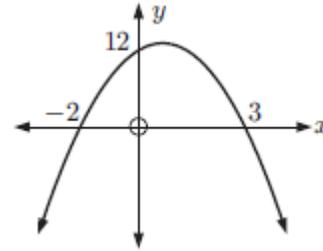
Ans:

b-



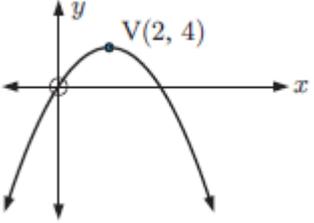
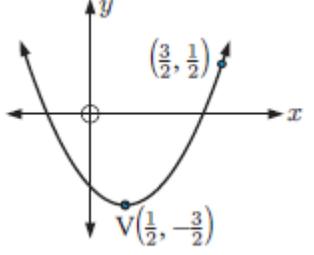
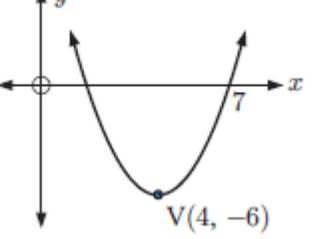
Ans:

c-

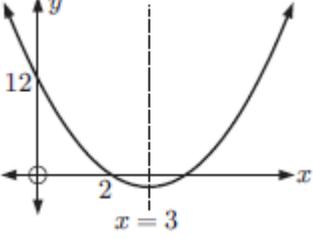
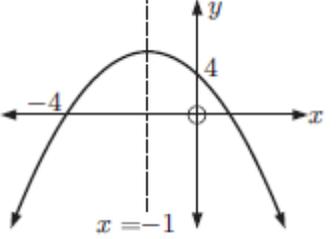
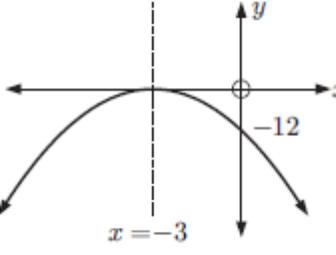


Ans:

2. If V is the vertex, find the equations of the quadratic function with graph:

<p>a-</p>  <p>Ans:</p>	<p>b-</p>  <p>Ans:</p>	<p>c-</p>  <p>Ans:</p>
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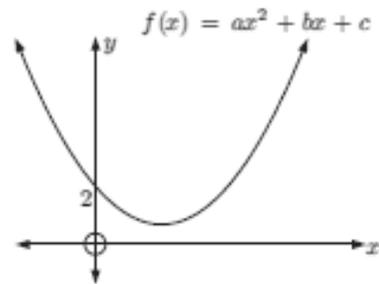
3. Find the equation of the quadratic with graph :

<p>a-</p>  <p>Ans:</p>	<p>b-</p>  <p>Ans:</p>	<p>c-</p>  <p>Ans:</p>
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4.

Consider the quadratic function $f(x) = ax^2 + bx + c$ alongside.

- State the value of c .
- The graph passes through $(1, 1)$ and $(2, 6)$. Use this information to write two equations in terms of a and b .
- Solve these equations simultaneously, and hence state the equation of the quadratic.



Ans:

a-

b-

c-

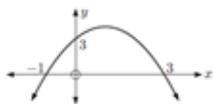
5.

The quadratic function $f(x) = ax^2 + bx + c$ has y -intercept -2 and axis of symmetry $x = 3$. The graph also passes through $(5, 3)$.

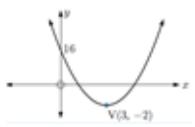
- State the value of c .
- Use the remaining information to write two equations in terms of a and b .
- Solve these equations simultaneously, and hence state the equation of the quadratic.
- Graph the quadratic using technology.

SLIDE SHOW

Find the equation of the quadratic with graph:



Find the equation of the quadratic with graph:



Find the equation of the quadratic with graph:



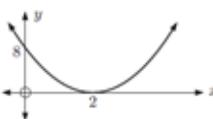
Summary:



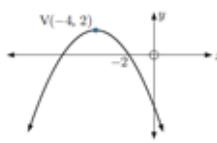
$$y = a(x - h)^2 + k$$

h, k is real

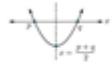
Find the equation of the quadratic with graph:



Find the equation of the quadratic with graph:



Summary:



$$y = a(x - p)(x - q)$$

p, q are real

Find the equation of the quadratic with graph:



Find the equation of the quadratic whose graph has the x-axis at $x = 2$ and $x = -3$, and which passes through the point $(1, -10)$. Give your answer in the form $y = a(x - p)(x - q)$.

Summary:



$$y = a(x - h)^2 + k$$