Date: April 17, 2014
Teacher: Tuğba Özcan
Number of Students: 18
Grade Level: 11 IB-SL
Time Frame: 45 minutes

Geometric Sequences

1. Goal(s)

- Students will be able to identify a simple geometric sequence and determine its formula both recursively and with an explicit formula.

2A. Specific Objectives (measurable)

- Students will be able to describe a simple geometric sequence
- Students will be able to generalize patterns of geometric sequences or determine its formula
- Students will be able to write geometric sequences both recursively and with an explicit formula
- Students will be able to find the *n*th term of a geometric sequence and the sum of a finite geometric sequence
- Students will be able to use a geometric sequence to solve an application problem

2B. Ministry of National Education (MoNE) Objectives

- İD.11.6.1.1. Genel terimi veya indirgeme bağıntısı verilen bir sayı dizisinin terimlerini hesaplar. Aritmetik, geometrik, kare sayı, üçgen sayı, Fibonacci vb. dizilerden örnekler verilir.

2C. NCTM-CCSS-IB or IGCSE Standards:

- Students should generalize patterns using explicitly defined and recursively defined functions (NCTM).
- Students should write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms (CCSS).
- Students should recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (CCSS).
- Students should identify a simple geometric sequence and determine its formula (IGCSE).

3. Rationale

- Students should recognize geometric sequences in everyday applications
- Students should recognize sequences that are not geometric
- Students will need to know the geometric sequences to apply their knowledge of geometric sequences to everyday life situations

4. Materials

- Board markers
- Computer and Projector
- Worksheets, exploration sheets (there are 18 students in class and copy each of them for 18 students)

5. Resources

- Hease and Herris Mathematics for the International Students SL
- http://arimaa.com/arimaa/links/chessStory.html

6. Getting Ready for the Lesson (Preparation Information)

- Copy worksheets for each student. There are 18 students in the class.
- Monitor students while they are working and help them if they need.

7. Prior Background Knowledge (Prerequisite Skills)

- Students will be expected to know the concept of pattern and concept of a sequence
- Students will be expected to familiar with basic number systems

Lesson Procedures

Transition: Hello Guys! Begin by asking students as "Have you ever wondered who invented chess?"Then tell the story of how chess was invented.

8A. Engage (5 minutes)

The story is following: Hundreds and hundreds years ago there was a King in India who loved to play games. But he had gotten bored of the games that were present at the time and wanted a new game that was much more challenging. He commissioned a poor mathematician who lived in his kingdom to come up with a new game. After months of struggling with all kinds of ideas the mathematician come up with the game of Chaturanga. The game had two armies each lead by a King who commanded the army to defeat the other by capturing the enemy King. It was played on a simple 8×8 square board. The King loved this game and so much that offered to give the poor mathematician anything he wished for. "I would like one grain of rice for the first square of the board, two grains for the second , four grains for the third and so on doubled for each of the 64 squares of the game board." said the mathematician. "Is that all?" asked the King. "Why don't you ask for gold or silver coins instead of rice grains?" ."The rice is sufficient for me ." replied the mathematician. The King ordered his staff to lay down the grains of rice and soon learned that all the wealth in his kingdom would not be enough to buy the amount of rice was exhausted before the 30th square reached."You have provided me with such a great game ant yet I cannot fulfill your simple wish. You are indeed a genius." said the King and offered to make the mathematician his top most advisor then.

Transition: Ask the students as "Are you wondering to know exactly how many grains of rice would be needed on the 64th square and how many total rice would be needed for all 64 squares?" Then tell them "Keep going to wonder! We will see together towards the end of this lesson."

B. Explore (10 minutes)

- Teacher distributes the exploration worksheet and gives students 10 minutes to work as a group of three.

Exploration Worksheet

The table shows the first four terms of a sequence.

Term 1	Term 2	Term 3	Term 4	Term 5	Term 6	Term 7
10	**20**	**40**	**80**			

1. Find the ratio between two consecutive terms.
2. What kind of relationship between the ratios that you have found?
3. Find the next three terms.
4. Try to find a short way to find the 20^{th} term.
5. By concerning the solution in 4, try to find the formula of the n^{th} term of the sequence.

- While students are studying as groups, teacher monitors them and help them if it is necessary. By the way evaluate them according to the rubric attached at the end of the lesson plan.

Transition: Ask students what they noticed in this sequence. Ask them as " Are there any pattern? What kind of pattern did you observed?"

C. Explain (10 minutes)

- After discussion, teacher explains what the geometric sequence is.

Definition: A sequence is *geometric* if there exists a number *r*. called the *common ratio*, such that

$$\frac{a_{n+1}}{a_n} = r, \quad \text{or} \quad a_{n+1} = a_n r, \quad \text{for any } n \geq 1.$$

i.e. If we start with a particular first term, and then multiply the same number successively, we obtain a *geometric sequence*

- By showing the examples of the sequences below, teacher asks students as "Are the sequences geometric or not? If it is, find the common ratio."

Examples:

1. 3, 6, 10, 15, …
2. 1, -2, 4, -8, …
3. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$
4. $a_n = 2\left(\frac{2}{3}\right)^{n-1}$

- Then, teacher denotes the common ratio by *r*, and write the first few terms:

a_1,

$a_2 = a_1 r$,

$a_3 = a_2 r = (a_1 r) r = a_1 r^2$

$a_4 = a_3 r = (a_1 r^2) r = a_1 r^3$

- Teacher asks students as "Do you state the relation between the power of *r* and the number of the term, *n* in each case?"
- After discussion teacher asks this time as "Do you a formula for the *n*th term of the geometric sequence?"
- Students may come up with the following result: The *n*-th term of a geometric sequence is given by $a_n = a_1 r^{n-1}$, for any $n \geq 1$.
- Teacher ask the students "
- Next, teacher (via classroom discourse) gets the students find the sum of the first *n* terms of a geometric sequence is given by

$$S_n = \frac{a_1(r^n - 1)}{r - 1}, \quad \text{for any } r \neq 1.$$

Transition: Tell the students " I am distributing the worksheet, make a group of three and solve these problems as a group."

D. Extend (15 minutes)

- Distribute the formula paper and then distribute the worksheets. (attached at the end of the lesson plan)
- Encourage whole students in the class to solve every problems on the worksheets
- Say the students " I am evaluating your performance in group work according to the rubric."(rubric is attached at the end of the lesson plan)

Transition: At the beginning of the lesson I told you a story. Now it's time to find the answers of the question. Let's try with your TI-Calculator

E. Evaluate (3 minutes)

- In addition to the abilities of the students with TI-Calculator, evaluate each students performance in group work by using the rubric while they are studying the worksheet (rubric is attached at the end of the lesson plan)
- Observe the students during the lesson and assess them according to their responses to the questions asked throughout the classroom discourse.

9. Closure & Relevance for Future Learning (2 minutes)

- Ask students is there any points not understood or any question.
- Then say them ”Ok, thank you so much for this enjoyable lesson”

10. Specific Key Questions:

- Have you ever wondered who invented chess? (application) (analysis)
- Are you wondering to know exactly how many grains of rice would be needed on the 64th square and how many total rice would be needed for all 64 squares? (comprehension)
- What kind of relationship between the ratios that you have found? (analysis)
- What did you notice in this sequence? (comprehension)
- Are there any pattern? What kind of pattern did you observed? (analysis)
- Are the sequences geometric or not? (knowledge)
- Do you state the relation between the power of r and the number of the term, n in each case?" (synthesis)
- Do you find a formula for the nth term of the geometric sequence?" (analysis)
- How can we generalize the way that we observed? (analysis)

11. Modifications

- If students cannot remember previous lesson, give them some clues.
- If students do not give answer to your questions, wait 20 seconds more.
- If the students cannot share any idea the teacher will wait twenty seconds more.
- If the students need more time while they solve the exercises the teacher will give one more minutes.
- If the students cannot solve the exercises the teacher will come back the definitions or previous examples and explain again
- If students confuse about the answer clarify it by another way.
- If they cannot solve the problems or do not understand, want another student to explain their friends how they used which method

Extension Worksheet

1. Determine if the sequence is geometric. If it is, find the common ratio.

a) -1, 6, -36, 216, ...	b) 4, 16, 36, 64, ...	c) -1, 1, 4, 8, ...

2. Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

a) $a_n=3^{n-1}$	b) $a_n=-2.5\cdot 4^{n-1}$

3. Given the recursive formula for a geometric sequence find the common ratio, the first five terms, and the explicit formula.

a) $a_n= a_{n-1}\cdot 2$ $a_1 = 2$	b) $a_n= a_{n-1}\cdot 5$ $a_1 = 2$

4. Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

a) $a_1 = 4,\ r = 5$	b) $a_4 = 25,\ r = -5$

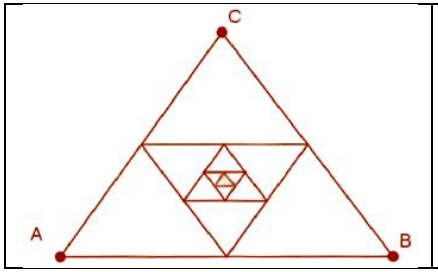
5. Given two terms in a geometric sequence find the 8th term and the recursive formula.

a) $a_4 = -12$ *and* $a_5 = -512$	b) $a_5 = 768$ *and* $a_2 = 12$

6. The number of pupils in a school increases at the rate of 5% per annum. When the school opened there were 200 pupils on the roll. How many pupils will there be in the school on the 10th anniversary of the opening?

7. A novice swimmer can complete the 50m breaststroke in 36s. He follows a programme designed to increasing improve his performance by 4% each year. How fast will he complete the 50m breaststroke after six years of the programme?

8. If the perimeter of the equiangular triangle ABC is *x* cm. The mid points of the sides are joined to form a smaller triangle and the process goes on as in the diagram. If the perimeter of the 6th triangle is 3cm. Find the value of *x*.



Rubric for Evaluation of Group Work

E: Excellent
G: Good
NI: Needs Improvement

	Student provided useful ideas and was a definite leader who contributed a lot of effort.			Student consistently focused on the task and what needed to be done. Very self directed.			Student almost always listened to, shared with and supported the efforts of others.			Student kept a positive attitude about the task.		
Student	**E**	**G**	**NI**	**E**	**G**	**NI**	**E**	**G**	**NI**	**E**	**G**	**NI**