Date: 12.12.2013
Teacher: Tuğba Özcan
Number of Students: 11
Grade Level: 10th Grade
Time Frame: 40 minutes

Discriminant and the Nature of the Quadratics

1. Goal(s)
- To develop concepts of the discriminant
- To identify the nature of roots of a quadratic equation

2A. Specific Objectives (measurable)
- Students will be able to realize the relationship between the value of discriminant and the number of real solutions(x-intercepts) for the quadratic equation
- Students will be able to learn determine the nature of the roots of a quadratic equation by using its discriminant
- Students will be able to identify three different situations of discriminant
- Students will practice evaluating the nature of roots of a quadratic equation by discriminant
- Students will be able to find the discriminant of the quadratic equations

2B. Ministry of National Education (MoNE) Objectives
- İkinci dereceden bir bilinmeyenli denklemin köklerini veren bağlantıyı gösterir ve köklerin varlığını diskriminantın işaretine göre belirler
- Parametre içeren ikinci dereceden bir bilinmeyenli bir denklemin köklerinin varlığını ve işaretini parametrelerin alacağı değerlere göre tablo üzerinde belirler
- Çözüm kümesi grafik ve cebir yardımıyla incelenir

2C. NCTM-CCSS-IB or IGCSE Standards:
- Students will be able to examine the discriminant to determine the relationship between the graph and x-axis.(IB)
- Students will be able to discuss quadratic equations by factorizing.(IB)
- Students will be able to understand the meaning of equivalent forms of expressions, equations, inequalities, and relations(NCTM algebra standard: represent and analyze)

3. Rationale

- The purpose of this lesson to teach solving quadratic equation by using the quadratic formula and determine whether the equation will have one or two or no solution
- The students will work on problems that require them to find the discriminant, nature of the roots, and the two solution using the quadratic formula
- Students need all information so they can analyze the graphs and how they related to given situations in the problems and they will reach a conclusion about the nature of the roots

4. Materials

- Exploration and extension worksheets for 11 students
- Worksheet for explanation part for 11 students
- A notebook in which there is a list of the names of the students and a pencil for assessment
- Two different colored board markers

5. Resources

- Haese &Harris Publications. Mathematics for the international student mathematics SL book. The authors are Robert Haese, Sandra Haese, Michael Hease, Marjut Maenpaa and Mark Humphries
- Haese &Harris Publications IGCSE Cambridge International Mathematics (0607) Extended book. The authors are Keith Black, Alison Ryan, Michael Haese, Robert Haese, Sandra Haese, and Mark Humphries.
- International mathematics for the middle years,MYP5 book, published by Pearson. The authors are Alan McSeveny, Rob Conway, Steve Wilkes, and Michael Smith
- 10.sınıf Esen yayınları. The authors are Nevzat Asma and Halit Bıyık
- T.C. MİLLİ EĞİTİM BAKANLIĞI Talim Terbiye Kurulu Başkanlığı Ortaöğretim Matematik (9,10,11 ve 12.sınıflar haftalık 4 saat) dersi öğretim programı
- MEB Ortaöğretim matematik 10. sınıf kitabı
- NCTMhttp://illuminations.nctm.org/Lesson.aspx?id=3016
- Math Forum http://mathforum.org/library/drmath/sets/select/dm_quadratic.html

6. Getting Ready for the Lesson (Preparation Information)

- Copy worksheets and extension papers for each student before the class. There are 11 students in the class.
- Explain students the instruction given in the worksheet.
- Let the students study worksheets individually or pearly
- Monitor the students while they are studying, and help them if they need

7. Prior Background Knowledge (Prerequisite Skills)

- Students will be expected to have algebraic thinking skills
- Students will be expected to know linear equation and the difference between linear equation and quadratic equation
- Students will be expected to be familiar with the factorizing a quadratic equation

Lesson Procedures

Transition: Hi guys! I am your mathematics teacher during one hour period. I am a trainee teacher from Bilkent University. I hope we will have a good lesson together.

8A. Engage (5-10min)

- Remind students that the topic of the lesson is the discriminant and the nature of the roots of the quadratic equations
- Write on the board the symbol of the discriminant and the formula of it as b^2-4ac=triangle and
- By showing the symbol ask students “Will you explain why b2- 4ac is triangle?”and “What do you think why the mathematicians represent this formula such a small triangle?” and wait them for a second to think about it and then talk about the story "This is all about how to solve equations such as: $40\ x^2 + 5\ x + 63 = 0$ right? People learned how to solve lots and lots of equations like this and then, a few hundred years ago, somebody asked what do the 40, the 5, and the 63 have to do with it? Can we find rules about what the answer is for every possible combination of the three "coefficients"? [In our example, 40 is the x^2-coefficient, 5 is the x-coefficient, and 63 is the constant term.] So one day, they set out to solve the equation: $ax^2 + b\ x + c = 0$. But the answers they got depended (obviously) on a, b and c, the coefficients. There were several different ways the problem could turn out, and, most interestingly, exactly which way it went was decided by this famous b^2 - 4ac. So they wanted to call that formula by a name, and the name that was decided was "discriminant," because it discriminated between the various solutions. But, as you know, in math we like to call things by single letters,

like a, b, f, x and so on. So we use the Greek letter Delta as its symbol. Delta looks exactly like a triangle, but it is a Greek letter, not a geometric shape. This letter is the big font d in the Greek alphabet. That is it actually represents d of the first letter of the word discriminant."

Transition: Why does it so we will observe and realize this at the end of the lesson. Now, try to answer the questions in the worksheet.

B. Explore (5-10 minutes)

- Distribute the worksheet(attached in this lesson plan)
- Give a clear instruction of the worksheet say "In this paper there are 6 exercises. I want you first to factorize the quadratic equations and find the roots of them and next find the their discriminant by using the discriminant formula."
- Wait 10 minutes for the student to solve the questions in the worksheet
- By the way walk around the students and learn them whether or not they need to help by asking "*Is there anything they are confused about?"*
- Walk around the students and take notes in your notebook about their assessments by observing or asking them questions such as " *Why do you think that?*", " *How did you reach that conclusion?*"

Transition: Say students to stop working and ask them "Did you notice anything in these examples, any pattern?" and wait for a second then after the discussion explain the pattern

C. Explain (10 min)

- Say "Ok, let's generalize how the delta change in which the root(s) exist or not."
- Make a table on the board and divide it 7 parts and write " equation, ax^2+bx+c form, a,b,c, the roots and the discriminant", respectively on top of them
- Then solve the quadratic equations on the worksheet which you distributed in the exploration part on the board by questioning method
- Write a quadratic equation and ask students "what are the coefficients of the equation?" and "What are the roots that you found? And how many roots are there?" and "What is the discriminant and is it greater than or equal to zero or less than zero?"
- Solve all 6 examples similarly on the board.
- By the way be sure all the students have the same solution by asking students "Does anyone disagree with this answer?" and discuss
- Write the situations on the table.
- Say students to draw the table on their notebooks

- Wait for students to write
- Next explain that if delta>0 then there are two real values for x; if delta=0 then there are two equal values for x; and if delta<0 then there will be no real solution.
- Hence say students “that i said at the beginning of the lesson, discriminant discriminate between the various solution.”

Transition: why there is no root when the delta is less than zero? What does this tell us? For what purpose we have already investigated the nature of the quadratic equations? And distribute the extension worksheet.

D. Extend (5 min)

- Explain the worksheet as “In this paper I want you to investigate the relationship between the nature of the roots and the sign of the discriminant
- Walk around the students and help them if they need or clarify the misunderstood points
- The worksheet attached the lesson plan

Transition: think about this proof of the discriminant and discuss with your friends in front of you or next to you

E. Evaluate (During the lesson)

- Say the students to write a single sentence about what they learn or understand from the proof after they shared their opinions with their friends
- Assess students’ knowledge and skills through oral questions
- Observe each student during the lesson and take notes about their assessment while they are studying the worksheets

9. Closure & Relevance for Future Learning

- Ask students if there are any points not understood.
- Then say that ”Ok, thank you so much for this enjoyable lesson”
- Give the students their homework
- State the next topic of the lesson

10. Specific Key Questions:

- How can you define the quadratic equations?(knowledge)
- How can you factorize a quadratic equation?(knowledge)
- What did you notice in this problem?(analysis)
- How can we generalize the way that we observed?(synthesis)

- Can you estimate what the pattern is?(evaluation)
- What if we chance the coefficients of the quadratic then What would the discriminant be?(application)
- Can you explain why the mathematicians use this symbol or why they called the formula discriminant?(comprehension)

11. Modifications
    - If the students cannot remember the factorization remind how to factorize
    - If the students cannot give answer your questions wait 20second more or give them clues
    - If students confuse about the answer clarify it by another way
    - If they cannot solve the problems or did not understand, want another student to explain their friends how she or he did or which method they used

Exploration worksheet:

Look at the examples of the quadratic equations below. First find the coefficients of each equation. Write them in the standard form of the quadratics as $ax^2+ bx+ c=0$. Then by factorization method, find the root(s) of the equations if exists. Finally find the discriminant of each equation by using the discriminant formula which is b^2-4ac.

1. $2x^2+5x-12=0$.

2. $X^2-4=0$

3. $9x^2-30x+25=0$

4. $49-14x+x^2=0$

5. $-3x^2+7x-11=0$

6. $X^2+x+5=0$

Extension worksheet:

Investigate the proof of the nature of the roots of the discriminant and then solve the questions and find the roots of the quadratic equations below by helping the proof.

The expression $ax^2 + bx + c$ is an interesting one. With some careful math, it can be split into two parts:

a x [varying part that's never negative] - [fixed part]

The fixed part is Delta/[4a], so you see the connection!

- If Delta is negative, the whole combination has absolutely no chance of being zero, so the equation $ax^2 + bx + c = 0$ cannot be solved. To see this, put the whole thing over a common denominator, and you get:

$$\frac{4a^2x \text{ [varying part, never negative]} - \text{Delta}}{4a}$$

- If Delta is negative, -Delta will be positive, and $4a^2$ is certainly positive, so the numerator is going to be at least as big as -Delta, so it can never be zero, regardless of the denominator.

- If Delta is positive, there will be two different values for x at which the whole expression becomes zero. This is the so-called generic case. To see this, I have to tell you that the "varying part" is just$(x + b/(2a))^2$, which takes any desired positive value exactly twice.

- If Delta is zero, the only value of x that will make the expression zero is x = -b/(2a), so you have one solution

1. $3x^2 -4x -1 = 0$

Δ=

2. $4 x^2 + 2x - 3 = 0$

Δ=

e.g. $x^2 -4x = 0$

a=1 so and if we write the equation as $x^2-4x+4-4=0$

The varying part that is never negative is x^2-4x+4. Or equally it is $(x-2)^2$ which is always greater than zero.

The fixed part is -4 which is equal to Delta/[4a]. i.e.[$(-4)^2-4*(1)*(0)]/4*(1)=4$