DEVELOPING ALGEBRAIC THINKING AND SENSE MAKING THROUGHOUT PROBLEM SOLVING

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Abstract

This article was designed to explain mathematical thinking with the help ofanalyzing problem solving. For a better understanding a problem, I used some mathematical problem solving techniques. Hence, throughout these articleI mentioned about these problem solving strategies and tested them on a problem, Eric the sheep. First, I drawn a picture to understand the problem. Then I explained my results making sense in the context of the problem. At the end , I reached a conclusion and described a general rule for the solution of the problem.

Key Words: problem solving, algebraic thinking, sense making, reasoning

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Introduction

The purpose of thisarticle was to state a problem, Eric the sheep, and make a solution of the problem. Also, throughout this problem I used mathematical problem solving strategies to understand the situation better. Hence, I explained my solution in this manner.

A Problem: Eric The Sheep

In 2009, NCTM published an important article entitled *"Focus in High School Mathematics: Reasoning and Sense Making".* In this article, it was emphasized that to promote mathematical thinking the various content areas should be used as tools. In this article, reasoning and sense making were defined as " the process of drawing conclusions on the basis of evidence or stated assumptions"(p.4) and " developing understanding of a situation, context or concept by connecting it with existing knowledge."(p.4), respectively.

For a better reasonıng and sense makıng in the mathematıcs classroom, students ability of mathematical thinking and being capable of solving a problem individually should be developed. According to Brahier, when teaching students to approach a problem, the students are supposed to examine the data to make conclusions and to connect the situations with the ideas that they have already developed(p.210). However, there are several ways to represent a problem, for example, it can be represented as a written rule, in words or symbols, as a graph, as an equation or as a table.

Now, I want to illustrate these more clearly in a problem, Eric the sheep. The problem is that " *It is a hot summer day, and Eric the sheep is at the end of a line of sheep waiting to be shorn. There are 50 sheep in front of him. Being impatient sort of sheep, though, every time the shearer takes a sheep from the front of the line to be shorn, Eric sneaks up two places in line. How many sheep will get shorn before Eric?"*In this problem, I started to draw a picture to visualize the line. I thought that Eric was at the end of the line and so he was the 51th sheep. If he sneaked up

two places in the line, he would be the 49th sheep. Then, if he sneaked up two places more in the line, he would be the 47th sheep and so on. Next, I wrote a rule so that there were 51 sheep in line and saying the getting shorn sheep in front of Eric would become the variable, x, and since Eric sneaked up two place in the line, I said2x to the rest of the line. After, I wrote an equation as 51-x=2x. hence, x would become 17.thus, 17 sheep would get shorn before Eric. Then, I checked my answer by writing all situations. That is, Eric was the 49th sheep when 1 sheep got shorn; then he was the 47th sheep when 2 sheep got shorn. It went on like this up to the 17th sheep got shorn so Eric was the 17th sheep. In this manner, I made a table. In this table, i wrote 1 if there was 1 sheep in front of Eric because only one sheep got shorn before Eric. Then, i wrote 1 if there were 2 sheep in front of Eric since 1 sheep got shorn before Eric. Again for 3 sheep in front of Eric, I wrote 1 because 1 sheep got shorn before Eric. Then, if there were 4,5,or6 sheep in front of Eric, 2 sheep would get shorn before Eric. Next, if there were 7,8 or 9 sheep in front of Eric, then 3 sheep would get shorn before Eric. That is, the number of sheep got shorn before Eric, repeated 3 times and in each 3th number , namely multiple of 3, the number of sheep got shorn before Eric, increased 1.

What if I changed the ruse of the problem. That is, the problem would be *"Eric gets more and more impatient. Explore how your rule changes if Eric sneaks past 3 sheep at a time? How about 4 sheep at a time?"*for these situations, again I made a table. I observed the rule first when Eric sneaked past 3 sheep at a time. If there was one sheep in front of Eric, then 1 sheep would get shorn before Eric. Next, if there were 4,5 or 6 sheep in front of Eric, then 1 sheep would get shorn. Next, if there were 5,6,7 or 8 sheep in front of Eric, then 2 sheep would get shorn before Eric. This pattern repeated in each 4 number and the number of sheep got shorn increased 1 in every number of multiple of 4. More clearly, for example, if there were 9 sheep in front of Eric, then 3 sheep would get shorn before Eric.

In the next situation which was that if Eric sneaked past 4 sheep at a time, the number of sheep got shorn before Eric changed in each 5th number and increased 1 in every number of multiple of 5. If I generalized the rule, I said that I had ***A*** number of sheep in front of Eric and Eric sneaked past ***n*** sheep at a time. Then, the number of sheep got shorn before Eric would be ***x+1*** where***x=A/n*** and for ***A is not equal to 0 in modulo n*** or the number of sheep got shorn before Eric would be ***x*** where ***x=A/n*** for ***A is equal to 0 in modulo n,*** where ***n is in the positive natural numbers.***

To conclude, to solve this problem above, chose the picture representation and then making the table as a problem solving strategy since I could understand better visually. Hence, first I had decided to describe the situation through pictures and then words and finally through symbols. I drawn a picture to see the pattern more clearly. Next, I tried to reach a conclusion and to find a pattern for the smallest number first and then for the bigger numbers. At the end, I found a general rule. Finally, for any number of sheep in front of Eric my rule worked.

Conclusion

The purpose of this article was to emphasize the importance of mathematical thinking and sense making in problem solving. I stated a problem, Eric the sheep. Then, I tried to draw a general picture and used different mathematical problem solving techniques. I learned that thinking algebraically in problem solving helped me to understand the problem and to solve them accurately. As Brahier said an emphasis on mathematical thinking could help to understand better and to retain mathematical ideas (p.210).

References

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