Date: March 26, 2014
Teacher: Tuğba Özcan
Number of Students: 18
Grade Level: 11
Time Frame: 70 minutes

Angles in a Circle

1. Goal(s)

- Students will be able to find any angles in a circle.

2A. Specific Objectives (measurable)

- Students will be able to describe and apply the properties of the central angle, the inscribed angle, the angle between two secants, the angle between a secant and a tangent, the angle between two chords and the angle between a chord and a tangent
- Students will be able to solve and prove statements involving inscribed angles and angles formed by chords, secants, and tangents.
- Students will recognize and find measures of angles formed by intersecting secants and tangents in relation to intercepted arcs.

2B. Ministry of National Education (MoNE) Objectives

- Bir çemberde merkez, çevre, iç, dış ve teğet-kiriş açıları açıklar; bu açıların ölçüleri ile gördükleri yayların ölçülerini ilişkilendirir.
- Çapı gören çevre açının 90° olduğu fark ettirilir.

2C. NCTM-CCSS-IB or IGCSE Standards:

- Students should use trigonometric relationships to determine lengths and angle measures (NCTM).
- Students should use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture (NCTM).
- Students should use geometric models to gain insights into, and answer questions in, other areas of mathematics (NCTM)

- Students should construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle (CCSS).

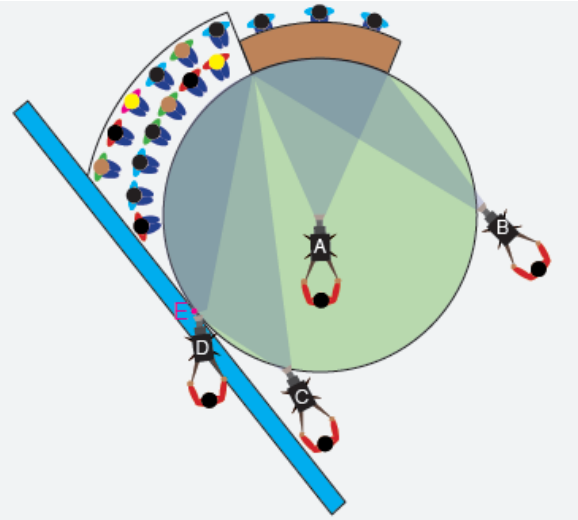
- Students should identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle (CCSS).

3. Rationale
   - Students should solve problems involving civil engineering, astronomy, and meteorology by the help of a knowledge of angle measures.
   - Students will need to know the relation between statistics and angles in a circle. Circle graphs are a useful way to display the data and they allow students to see important characteristics of the data at a glance.
4. Materials
   - Board markers
   - Computer and Projector
   - Worksheets, exploration sheets (there are 18 students in class and copy each of them for 18 students)
5. Resources
   - Koç School geometry documents
   - Glencoe Geometry Book
   - Hease and Herris Mathematics for the International Students SL and HL
   - Hease abd Herris Cambridge International Mathematics
6. Getting Ready for the Lesson (Preparation Information)
   - Copy worksheets for each student. There are 18 students in the class.
   - Before the lesson, open the projector and show the GSP.
   - Monitor students while they are working and help them if they need.
7. Prior Background Knowledge (Prerequisite Skills)
   - Students will be expected to state and apply the basic properties of circles
   - Students will be expected to familiar with terms such as arcs, tangents.

Lesson Procedures

Transition: Hello Guys! I am coming from Bilkent University and I will be teaching this lesson. Today, we're going to study angles in circles.

8A. Engage (5 minutes)

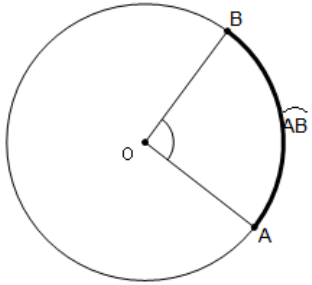


- Show the picture above and tell the students "In this picture you are seeing a circumferential television studio. The cameras A, B, C , D are recording the table and the audiences in certain angles."
- Then tell the students " Look at the cameras, which are A and B, recording the table. And then compare their visual angle."
- " Since the circumferential television studio is tangent to the wall in the point E, identify the rays which construct the angle of the camera C."
- "Compare the visual angles of the cameras C and D which see the same arc. "
- " When the camera C is recording the arc of tables, what is the relationship between the visual angles of the cameras A and C?"

(Do the explanation part with the exploration part. That is, solve an example after each property given in the GSP exploration part. Give at least one example for every property.)

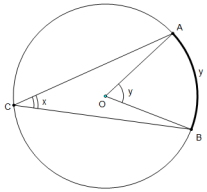
Transition: Let's discover how we can find the measures of angles in circles.
B. Explore (20 minutes)
AND
Transition: Let's see the applications of these properties.
C. Explain (10 minutes)

- Open the GSP and meanwhile distribute the exploration sheet which is attached at the end of the lesson plan.

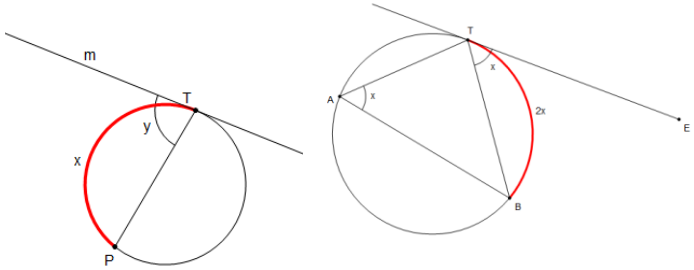


- Open the "Arc" property in GSP. Show central angle and arcs above. Then move the points A and B. First sketch 360 °, then 180° and ask the students that " What is the arc seeing the angle < *AOB?* What is the measure of the arc AB?"
- Then wait for the students' responses and discuss with them. And construct any angle by carrying the point A.
- Ask the students "What if I move the point A to anywhere, let the angle be θ, then what is the arc seeing the angle < *AOB?* and What is the measure of the arc AB?" At the end of the discussion show the conclusion which is $m(\angle AOB) = m(\overset{\frown}{AB})$

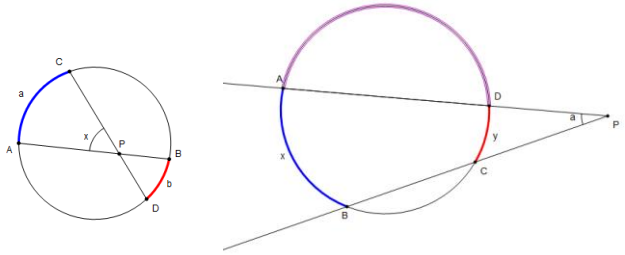
- Secondly, open the GSP and show "Theorem 1" which is related with the inscribed angle.



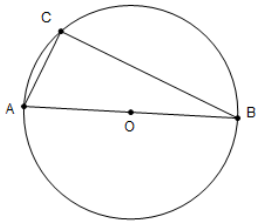
- Again do not give directly the proof of the theorem. Instead ask the students some questions and get the students explore the property of the theorem.
- In similar way, give the other properties and theorems in GSP. Give "Theorem 2 and Theorem 8", below and get the students explore the proof of the theorems by asking them questions as previous ones.



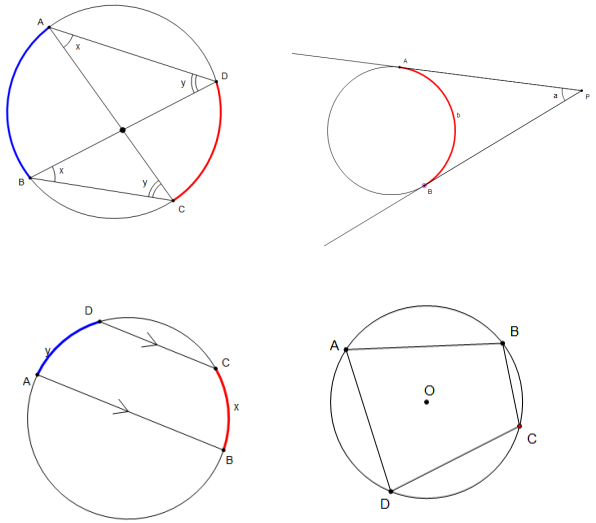
- Give "Theorem 3 and Theorem 4", in GSP. Get the students explore the proof of the theorems by asking them questions as previous ones.



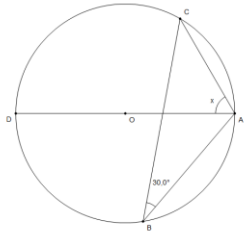
- Give "Theorem 7", in GSP. Get the students explore the proof of the theorems by asking them questions as previous ones

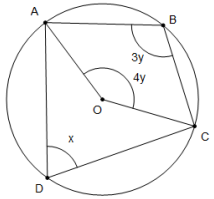


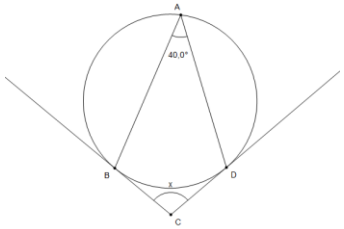
- Give "Theorems 9, 11, 13, 14", in GSP. Get the students explore the proof of the theorems by asking them questions as previous ones



- Give example below after properties and show the solution in GSP.







Transition: Tell the students " I am distributing the worksheet, make a group of four and solve these problems as a group."

D. Extend (30 minutes)

- Distribute the formula paper and then distribute the worksheets. (attached at the end of the lesson plan)
- Encourage whole students in the class to solve every problems on the worksheets
- Say the students " I am evaluating your performance in group work according to the rubric."(rubric is attached at the end of the lesson plan)

Transition: I am evaluating your performance in group work according to the rubric

E. Evaluate (during the lesson and in extend part)

- Evaluate each students performance in group work by using the rubric while they are studying the worksheet (rubric is attached at the end of the lesson plan)
- Observe the students during the lesson and check each student's answer.

9. Closure & Relevance for Future Learning (5 minutes)

- Ask students is there any points not understood or any question.
- Then say them "Ok, thank you so much for this enjoyable lesson"

10. Specific Key Questions:

- What is the relationship between an angle and its intercepted arcs when the angles vertex is on the circle, inside the circle and outside the circle? (analysis)

- When the camera C is recording the arc of the place of the table, how will be the relationship between the angles of visions of the cameras A and C?
- What is the arc seeing the angle < *AOB?* What is the measure of the arc AB? (knowledge)
- What if I move the point A to anywhere, let the angle be θ, then what is the arc seeing the angle < *AOB?* and What is the measure of the arc AB? (application)
- How can you define sin of an angle in a right triangle? (knowledge)
- What did you notice in this theorem? (analysis)
- How can we generalize the way that we observed?(synthesis)

11. Modifications

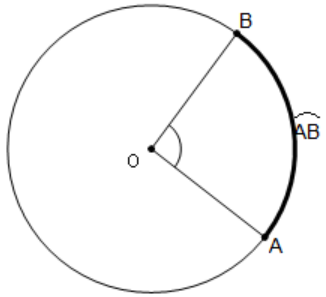
- If students cannot remember previous lesson, give them some clues.
- If students do not give answer to your questions, wait 20 seconds more.
- If the students cannot share any idea the teacher will wait twenty seconds more.
- If the students need more time while they solve the exercises the teacher will give one more minutes.
- If the students cannot solve the exercises the teacher will come back the definitions or previous examples and explain again
- If students confuse about the answer clarify it by another way.
- If they cannot solve the problems or do not understand, want another student to explain their friends how they used which method

Exploration Sheet 1

Angles in a Circle

Theorems

1.



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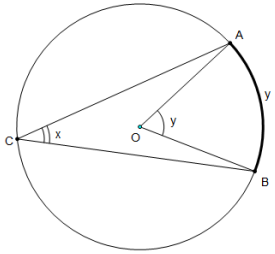

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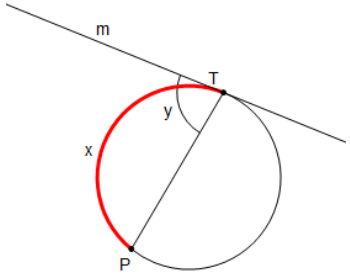

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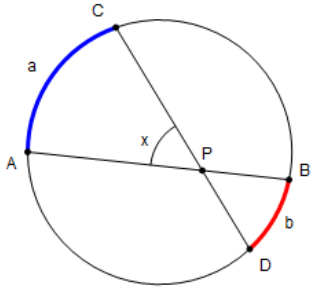

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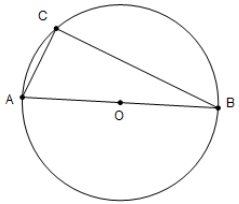

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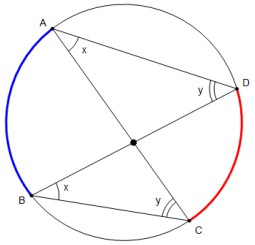

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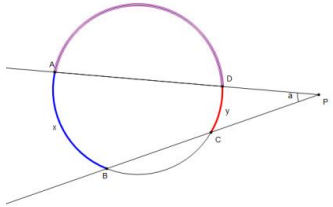

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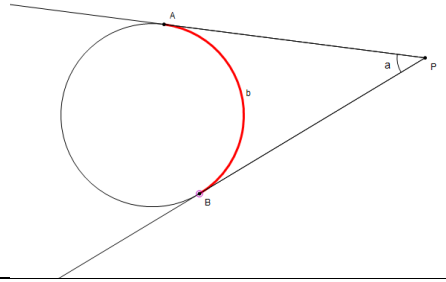

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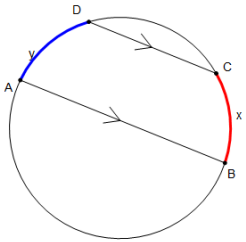

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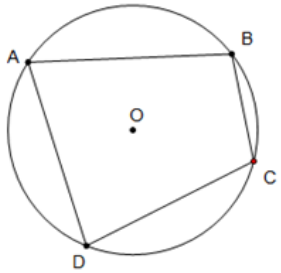
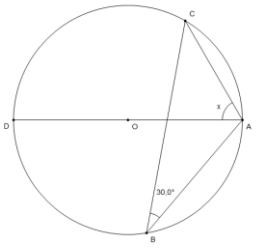


Exploration Sheet 2

Angles in a Circle

Questions

1.



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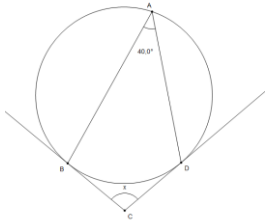

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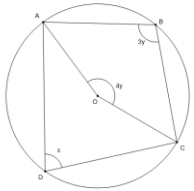

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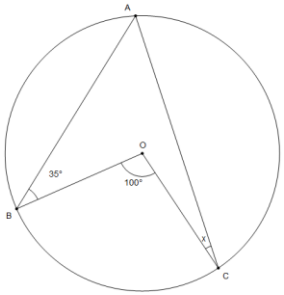

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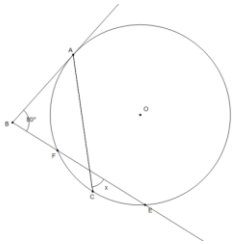
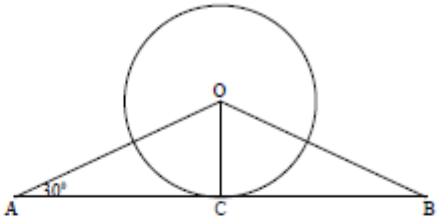


Worksheet

Angles in a Circle

1. Triangle AOB is isosceles.
AB is a tangent to the circle.
Angle CAO is 30^0.

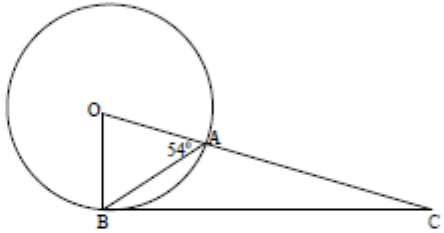
Calculate the size of angle BOC.



2. In the triangle opposite

OB is a radius of the circle
BC is a tangent to the circle
Angle OAB = 54^0.

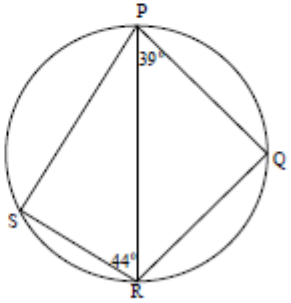
Calculate angle BCA.



3. PR is a diameter of the circle.

Angle PRS is 44^0
Angle QPR is 39^0.

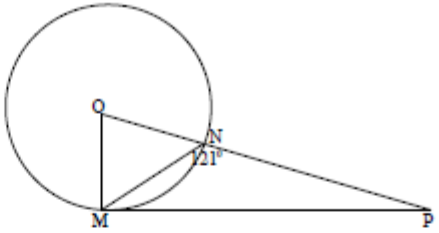
Calculate the size of angle SRQ.



4. In the diagram

OM is a radius of the circle
MP is a tangent to the circle
Angle MNP = 121^0

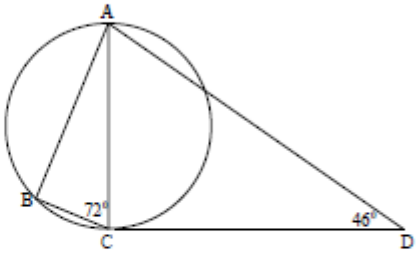
Calculate angle MPN.



5. AC is a diameter of the circle.
CD is a tangent to the circle.

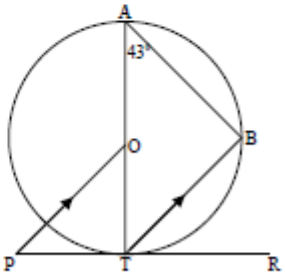
Angle ACB = 72^0.
Angle CDA = 46^0.

Calculate the size of angle DAB.



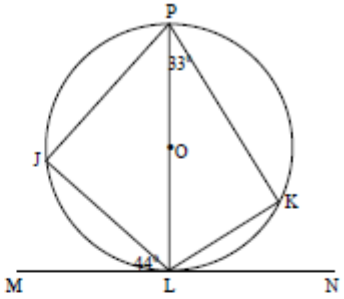
6. PTR is a tangent to the circle, centre O.
Angle BAT = 43^0.
PO is parallel to TB.

Calculate the size of angle OPT.



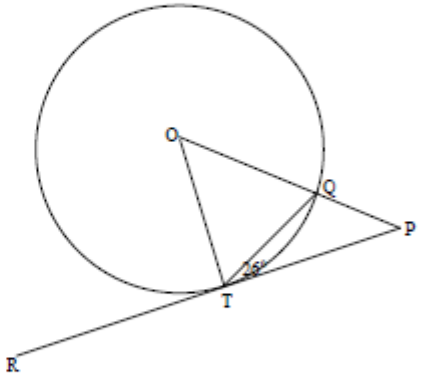
7. MLN is a tangent to the circle, centre O.
Angle JLM is 44^0.
Angle KPL is 33^0.

Find the size of angle KLJ.



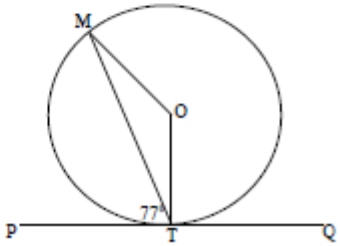
8. RP is a tangent to the circle, centre O.
Angle QTP is 26^0.

Calculate the size of angle OPT.



9. PTQ is a tangent to the circle, centre O.
Angle MTP = 77^0.

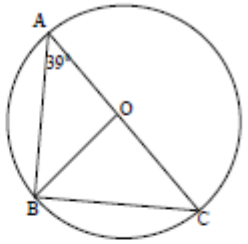
Calculate the size of angle MOT.



10. In the diagram O is the centre of the circle.

AC is a diameter.
B is a point on the circumference.
Angle BAC = 39^0.

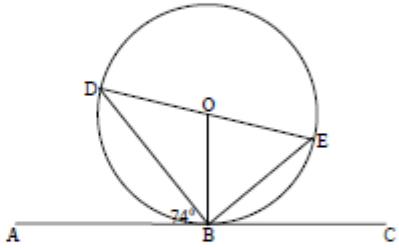
Calculate angle BOC.



11. The diagram shows a circle centre O.
AC is a tangent to the circle.

Angle DBA is 74^0.

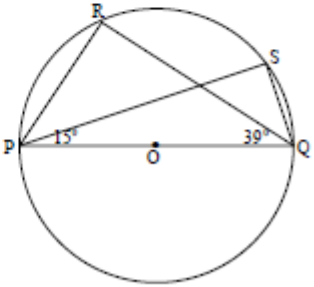
Calculate the size of angle BOE.



12. PQ is a diameter of the circle, centre O.
R and S are points on the circumference.

Angle SPQ is 15^0.
Angle RQP is 39^0.

Calculate the size of angle RPS.



Scoring Rubric-Cooperative Group Work
Criteria: Excellent/Good/ Needs Improvement

E	G	N I	Contributions:	Comments:
			Provides useful ideas. A definite leader who contributes a lot of effort.	
			Consistently stays focused on the task and what needs to be done. Very self directed.	
			Almost always listen to, shares with, and supports the efforts of others.	
			Keeps a positive attitude about the task.	