

TI Calculator Worksheet

Grade 11

Subject: Solving System of Linear Equations by Using Matrices

Necessary Equipments: TI-84 Plus Calculator, Computer, Excel Programming

Time Required: 60 minutes

Note: Remember! In TI-84 Plus Calculator, minus and negative signs(-) are different from each other. If you press the minus button instead of the negative sign you will get syntax error.

Objectives:

- İD. 11.3.2.2.İkinci dereceden iki bilinmeyenli denklem sistemlerinin çözüm kümesini cebir ve grafik yardımıyla bulur.

History of Matrices

- The beginnings of matrices and determinants goes back to the second century BC although traces can be seen back to the fourth century BC.
- It is not surprising that the beginnings of matrices and determinants should arise through the study of systems of linear equations. The Babylonians studied problems which lead to simultaneous linear equations and some of these are preserved in clay tablets which survive. For example a tablet dating from around 300 BC contains the following problem:
 - *There are two fields whose total area is 1800 square yards. One produces grain at the rate of $\frac{2}{3}$ of a bushel per square yard while the other produces grain at the rate of $\frac{1}{2}$ a bushel per square yard. If the total yield is 1100 bushels, what is the size of each field.*

Matrix

A way of representing data in a rectangular array. An $m \times n$ matrix has m rows and n columns, and each entry is given a unique name, based on its row and column: The matrix A is often denoted $[A]$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Properties of Matrices

- **Dimensions** - The number of rows and the number of columns in a matrix. For example, a 3×4 -dimensional matrix has 3 rows and 4 columns.
 - Here is an example of a 3×2 -dimensional matrix:

$$A = \begin{bmatrix} 1 & 16 \\ -12 & -\frac{3}{2} \\ 4 & 0 \end{bmatrix}$$

The number a_{12} is the number in the **1st row** and the **2nd column**. Thus,

$a_{12} = 16$. a_{21} is the number in the **2nd row** and the **1st column**. Thus,

$$a_{21} = -12.$$

- **Identity Matrix** - The matrix which, when multiplied by any matrix, does not change the matrix. In other words, any matrix multiplied by I is equal to itself.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- **Determinant of a Matrix** - The determinant of a matrix is a scalar property of that matrix, which can be thought of physically as the volume enclosed by the row vectors of the matrix. Only square matrices have determinants. Determinants are also useful because they tell us whether or not a matrix can be inverted

Activity 1: Find the determinant of a square matrix by using TI calculator.

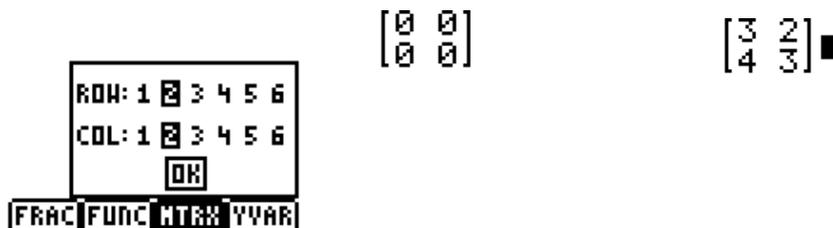
1. Press **ALPHA [F3]** to display the quick matrix editor. The default size of the matrix is two rows by two columns. But you can choose any dimensional matrix.

2. Press **ENTER**.

3. For example write a 2 by 2 matrix which is $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$. So press **3** \blacktriangleright **2** \blacktriangleright **4** \blacktriangleright **3** to

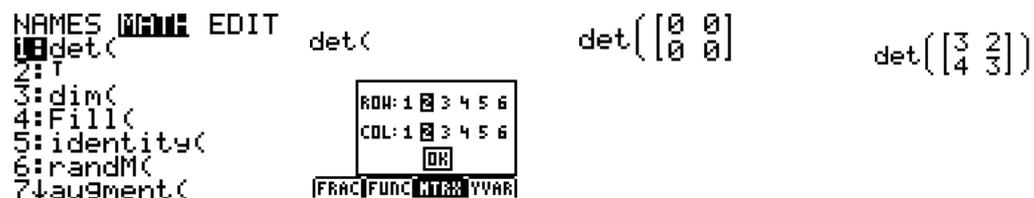
create the matrix.

Note: To input a fraction in a matrix, delete the pre-populated zero first.



4. To find the determinant of the matrix given; Press To display the **MATRIX MATH** menu, press **2nd MATRIX** \blacktriangleright . Choose **1: det (** and press **ENTER**.

5. Press **ALPHA [F3]** to display the matrix $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$. Then, press **ENTER**.



Test yourself!

1. The area of a parallelogram with one vertex at the origin can be computed by finding the determinant of a matrix having the coordinates of the vertices adjacent to the origin. Consider a parallelogram with vertices $(0, 0)$, $(3, 1)$, $(2, 5)$, and $(5, 6)$. The vertices adjacent to the origin are $(3, 1)$ and $(2, 5)$. Find the area of the parallelogram. Use TI calculator.

Note: *The determinant may be positive or negative depending on the order in which coordinates are entered in the matrix. Of course, the area is always positive.*

2. A parallelepiped (the three-dimensional analog of the parallelogram) has one vertex at the origin and the three vertices adjacent to the origin at $(1, 2, 3)$, $(1, 3, 4)$, $(1, 3, 5)$. The volume of the figure is 1. Verify that the appropriate determinant gives the same value for the volume. Use TI calculator.

- **Inverse of a Matrix**

The multiplicative inverse of a real number is the number that yields 1 (the identity) when multiplied by the original number. $\frac{1}{a}$ is the multiplicative inverse of a , because $a \times \frac{1}{a} = 1$.

Most matrices also have a multiplicative inverse. In other words, for the majority of matrices A , there exists a matrix A^{-1} such that $AA^{-1} = I$ and $A^{-1}A = I$

Activity 2: Find the inverse of the matrix above by using TI calculator.

Be careful! In order inverse of a matrix be exist, the matrix must be a square matrix and determinant of it cannot equal zero.

Note: Remember! In TI-84 Plus Calculator, minus and negative signs(-) are different from each other. If you press the minus button instead of the negative sign you will get syntax error.

1. Press $\mathbf{x^{-1}}$ to invert a matrix. Next, press **ALPHA [F3]** to display the matrix

$$\begin{pmatrix} 4 & 2 & 1 \\ -2 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

2. Then, press **ENTER**.

$$\text{Ans}^{-1} \begin{bmatrix} 4 & 2 & 1 \\ -2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \blacksquare \quad \text{Ans}^{-1} \begin{bmatrix} 4 & 2 & 1 \\ -2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 1 \\ -2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Test yourself!

1. Find the inverse of the matrix $\begin{pmatrix} 3 & -3 & 4 \\ 7 & -1 & -3 \\ -6 & 5 & 0 \end{pmatrix}$

- **Multiplication of Two Matrices**

Activity 3: Find the multiplication of the matrices below by using TI calculator.

$$\begin{bmatrix} 1 & 6 & -2 \\ 0 & -3 & 10 \end{bmatrix} \begin{bmatrix} 5 & 0 & -1 & 1 \\ 2 & 4 & 0 & 6 \\ -1 & -2 & 4 & \frac{1}{2} \end{bmatrix} = ?$$

1. To multiply two matrices together, the column dimension of matrix A must match the row dimension of matrix B. Press **ALPHA [F3]**.
2. First, for matrix A, the default size of the matrix is **two rows** by **three columns**.
3. After writing the first matrix A, press \times . Then, press **ALPHA [F3]**.
4. And, for matrix B, the default size of the matrix is **three rows** by **four columns**.

5. Finally, press **ENTER**.

$$\begin{bmatrix} 1 & 6 & -2 \\ 0 & -3 & 10 \end{bmatrix} * \begin{bmatrix} 1 & 6 & -2 \\ 0 & -3 & 10 \end{bmatrix} * \begin{bmatrix} 5 & 0 \\ 2 & 4 \\ -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 19 & 28 & -9 & 36 \\ -16 & -32 & 40 & -13 \end{bmatrix}$$

Thus, the answer is

$$\begin{bmatrix} 19 & 28 & -9 & 36 \\ -16 & -32 & 40 & -13 \end{bmatrix} \text{ a } 2 \times 3 \text{ matrix times a } 3 \times 4 \text{ matrix is}$$

a 2×4 matrix.

Note that *Matrix multiplication is not necessarily commutative: it is not always true that $AB = BA$. (See Appendix 1 for more information about commutative property)*

Note : *Multiplying by the identity matrix I doesn't change anything, just like multiplying by number 1 doesn't change anything. (See Appendix 2 for more information about multiplying by the identity.*

Test yourself!

1. Find the multiplication of the matrices $A = \begin{pmatrix} 0 & 2 & 4 \\ 6 & 8 & 0 \\ 2 & 4 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & 5 \\ 7 & 9 & 1 \\ 3 & 5 & 7 \end{pmatrix}$

Find AB and BA . Use TI calculator.

System of Linear Equations

A system of equations is a collection of two or more equations with a same set of unknowns.

In solving a system of equations, we try to find values for each of the unknowns that will satisfy every equation in the system.

- **Reduced Row-Echelon Form** - The form of a matrix in which the first $m \times m$ entries from the identity matrix:

$$\begin{bmatrix} 1 & 0 & 0 & * & * & \cdots \\ 0 & 1 & 0 & * & * & \cdots \\ 0 & 0 & 1 & * & * & \cdots \end{bmatrix}$$

Activity 4 :

The air-mail rate for letters to Europe is 45 cents per half-ounce and to Africa as 65 cents per half-ounce. If Shirley paid \$18.55 to send 35 half-ounce letters abroad, how many did she send to Africa?

We can translate the word problem into an algebraic equations as :

$$0.45x + 0.65y = 18.55$$

$$x + y = 35$$

Hence, we can convert the system to a matrix equation. That is,

$$\begin{pmatrix} 0.45 & 0.65 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18.55 \\ 35 \end{pmatrix}$$

By multiplying both sides of the equation by the inverse of the matrix $\begin{pmatrix} 0.45 & 0.65 \\ 1 & 1 \end{pmatrix}$

which is represented by $\begin{pmatrix} 0.45 & 0.65 \\ 1 & 1 \end{pmatrix}^{-1}$.

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.45 & 0.65 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 18.55 \\ 35 \end{pmatrix}$$

a) Solving system of linear equations by using TI calculator

Thus, use TI calculator and;

1. Press **2nd** **MATRIX**. Press $\triangleright \triangleright$ to display the **MATRIX EDIT** menu. Press **1** to select **1:[A]**.

```
NAMES MATH [0.00]
1: [A]
2: [B]
3: [C]
4: [D]
5: [E]
6: [F]
7↓ [G]
```

2. Press **2** **ENTER** **2** **ENTER** to define a **2×4** matrix. The rectangular cursor indicates the current element. Ellipses (...) indicate additional columns beyond the screen.

```
MATRIX[A] 2 ×■
[ 0      0      ]
[ 0      0      ]
```

3. Press **0.45 ENTER** to enter the first element. The rectangular cursor moves to the second column of the first row. Press **0.65 ENTER 1 ENTER 1 ENTER** to complete the first and second row

```
MATRIX[A] 2 ×2
[ 0.45      0.65      ]
[ 1         1         ]
```

2, 2=1

4. Press **2nd MATRIX**. Press **▷ ▷** to display the **MATRIX EDIT** menu.

Press **2** to select **2: [B]**. Press **2 ENTER 1 ENTER** to define a **2×1** matrix. The rectangular cursor indicates the current element. Press **18.55 ENTER 35 ENTER**

```
MATRIX[B] 2 ×1
[ 18.55      ]
[ 35         ]
```

5. Press **2nd Quit** to return to the home screen. If necessary, press **CLEAR** to clear the home screen. Press **2nd MATRIX** and choose **1: [A]** and press **ENTER**.

```
MATRIX MATH EDIT
1: [A] 2×2
2: [B] 2×1
3: [C]
4: [D]
5: [E]
6: [F]
7↓ [G]
```


3. Choose all four cells of the inverse of the coefficient matrix and write "=" and then start writing "**minver..**" which means that the inverse of the matrix. So you will see the function **MINVERSE**. Thus, double click on that and you will notice the open parenthesis. Choose the cells on which the coefficients of the x and y- variables.

4. Now, you will see the cell numbers of the coefficient matrix. For example, considering the above excel paper, the coefficient of x-variable of the first equation, the coefficient 0.45 is represented by B3 where B is the column number and 3 is the row number.

	A	B	C	D	E	F	G	H	I
1		Coefficient matrix				Right hand side			Solution
2		x	y						
3	Eqn. 1	0.45	0.65			18.55		x	
4	Eqn. 2	1	1			35		y	
5									
6		Inverse of Coefficient Matrix							
7		=minverse(B3:C4							
8									
9									

5. Do not close the parenthesis. Instead, press the buttons **SHIFT+CTRL+ENTER** simultaneously. So, you will obtain the inverse matrix.

6. Now, draw a 2×1 box which represents the number of constant term. You can label this box as "Solution". Then, choose these solution cells and write "=" and write "**mmult**" so that choose "**MMULT**" function that means matrix multiplication.

Double click on that . Again you will see an open parenthesis. First choose the entire cells of the inverse of coefficient matrix; and put comma and choose the cells of the constant terms. Again, press the buttons **SHIFT+CTRL+ENTER** simultaneously. So, you will obtain the x and y values.

Test yourself!

1. Solve the systems of equations in TI calculator and in Excel Programming.

a) $x + 2y = 3$

$3x + 8y = 1$

b) $2x + 4y + 8z = 6$

$x + 3y + 5z = 4$

$3x + 8y + 6z = 19$

2. A delivery company with offices in three cities distributes four types of equipment to its clients: typewriters, computers, copy machines, and printers. The number of units shipped from each office during the month of October is shown in the table below.

	Office		
	A	B	C
Typewriters	73	69	100
Computers	80	41	82
Copy Machines	30	13	41
Printers	150	120	90

Profit on a single unit is: typewriter \$30, computer \$42, copy machine \$65, printer \$27. Find the amount of profit made by each office for the month of October.

Activity 5: Find the solution of the system of linear equation below by row reduced echelon method..

$$(1) \quad x + 2y = 3$$

$$(2) \quad 2x + 3y = 3$$

Start with writing the system of equation as $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix}$ where the first column

represents the coefficients of the x variable in the system, the second column represents the coefficients of the y variable in the system, and the third column represents constant values.

Thus, use TI calculator and;

1. Press **2nd** **MATRIX**. Press $\triangleright \triangleright$ to display the **MATRIX EDIT** menu.

Press **1** to select **1: [A]**.

```

NAMES MATH [EDIT]
1: [A]
2: [B]
3: [C]
4: [D]
5: [E]
6: [F]
7↓ [G]

```

2. Press **2** **ENTER** **3** **ENTER** to define a **2×3** matrix. The rectangular cursor indicates the current element. Ellipses (...) indicate additional columns beyond the screen.

```

MATRIX[A] 2 × 3
[ 0  0  0 ]
[ 0  0  0 ]

```

```

1, 1=0

```

3. Press **1 ENTER** to enter the first element. The rectangular cursor moves to the second column of the first row.

4. Press **2 ENTER 3 ENTER** to complete the first row for (1).

```
MATRIX[A] 2 ×3
[ 1  2  3 ]
[ 0  0  0 ]
```

```
2, 1=0
```

5. Press **2 ENTER 3 ENTER 3 ENTER** to enter the second row for (2).

```
MATRIX[A] 2 ×3
[ 1  2  3 ]
[ 2  3  0 ]
```

```
2, 3=3
```

6. Press **2nd MATRIX** \triangleright to display the **MATRIX MATH** menu. Select **B:rref()** to copy **rref()** to the home screen.

```
NAMES [ ] EDIT
6:randM(
7:augment(
8:Matr▶list(
9:List▶matr(
0:cumSum(
A:ref(
B:rref(
```

7. Press **2nd MATRIX 1** to select **1: [A]** from the **MATRX NAMES** menu. Press **▷**
ENTER. The reduced row-echelon form of the matrix is displayed and stored in **Ans**.

```
rref([A]■          rref([A])  
                    [1 0 -3]  
                    [0 1 3]
```

∴ That is, $x = -3$ and $y = 3$.

Test yourself!

First use row reduced echelon matrix method for solving the problem below by using TI calculator. Then, solve the same problem by using Excel Programming and compare your results. Write the similarities of these two tools in matrices.

A cash drawer contains 95 bills totaling \$960. The number of \$10 bills is 6 more than 4 times the number of \$20 bills. The number of \$5 bills is 2 less than 2 times the number of \$20 bills. How many bills of each denomination are in the cash drawer?

REFLECTION

If someone asked me ten years ago as " How do you learn a new thing ?", probably I would answer the question like "I take a book, read it and take some notes. And I believe the best learning could become with the paper and pen." However, if someone asked me this question now, I would say that learning is like a sea itself which depends on how large your horizon is. What am I saying is that I can learn about learning. Precisely at this point, preparing this kind of worksheet broaden my perspective to learn about learning.

While I was preparing the worksheet, I realized that developing technology was such a significant learning way that it facilitated our life provided it was used properly and for a significant purpose. For instance, although it was known and seen as very hard to learning mathematics, I noticed that this could be helpful for people who were bias about mathematics. Since, using technology made something easy and enjoyable to learn. On the other hand, technology like TI calculator and Excel programming could get people visualize mathematics, experience it and even get them like math. I would not know how enjoyable and easy to solve matrices by TI calculator or Excel before trial. Since the unit of matrices was a huge topic and you could not believe that almost all problems can be solved by matrices even though solving matrices by hand quite difficult. However, technology came to our life fortunately. I love matrices more now.

REFERENCES

<http://www.purplemath.com/modules/mtrxmult3.htm>

<http://www-groups.dcs.st->

[andrews.ac.uk/~history/HistTopics/Matrices_and_determinants.html](http://www-groups.dcs.st-andrews.ac.uk/~history/HistTopics/Matrices_and_determinants.html)

<http://phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=ajd&wcsuffix=1301>

<http://phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=aga&wcsuffix=0452>

<http://www.hsrc.org/>

http://www.frtr.gov/matrix2/top_page.html

<http://mathforum.org/library/drmath/view/55464.html>

<http://www.youtube.com/watch?v=gSNa3fQX0WQ>